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GENERAL DESCRIPTION OF THE MOON.

Part of the introductory chapter from "A Comparison of the Features of the Earth and the Moon," by Prof. N. S. Shaler, Smithsonian Contributions to Knowledge, Vol. XXXIV (No. 1438), 1903. Quarto, pp. 78, with 25 plates.

The history of primitive astronomy shows that the moon, of all celestial objects, from the beginning of man's intellectual development has been the most closely observed. Although the sun was doubtless recognized by the lowliest man as the most important feature of the heavens, as the giver of life, the conditions under which it is seen, especially its blinding light, long made any extended study of it impossible. So, except for the very evident changes of its course across the sky and the consequent succession of the seasons, little was known of the solar center two hundred years ago, and, save its approximate distance from the earth, its mass, and its general relations to the planets, not much knowledge of it was gained until the last century. On the other hand the moon, because of its nearness, being only about one four-hundredth part as remote from the earth as the sun, has in a noteworthy way entered into the records of men. Its relatively short period of change and the very pronounced character of its alterations made it the first index of time beyond the round of the day. It is evident, indeed, that as soon as men began to reckon time they used the lunar month to make their tally, rather than that of the solar year. Moreover, the surface of the moon reveals much to the naked eye, not clearly, but sufficiently well to afford the basis for speculation and to tempt the imagination to create there a world like our own. It is therefore not surprising that a host of myths concerning the nature of our satellite grew up in the days before the telescope. It is interesting to note the fact

PLATE I.

Age of moon, 8 days 4 hours. September 22, 1890. Lick Observatory.

(In accordance with the usage of selenographers, the plates are printed in the reversed order in which they appear in a celestial telescope. The top of each is the south, the bottom the north, the right-hand the east, and the left the west.)

In Pl. I the most noteworthy features are the maria of the western half of the visible portion of the sphere. The rudely circular form of these fields is well shown, also the fact that none of them extend to the margin or "limb" of the moon. The bright, slightly curved ridge in the lower half of the picture facing the partly illuminated mare—the Mare Imbrium—is the apennines; the large vulcanoid at its southern end is Eratosphenes. The larger pit in the ocean opposite the center of the range is Archimedes; the two craters next to the north are, the nearer, Autolycus, and the farther and larger, Arispillus. The larger of the two dark pits near the northern end of the Apennines is Eudoxus, the smaller Aristoteles. Southeast from these craters lie the Alps, a group of bright peaks extending in a northeast and southwest direction. A faint, dark streak shows the position of the Alpine valley. The flat, irregular area north of the range is the M. Frigoris.

Close inspection of this plate will show that many of the vulcanoids (*a*) have pits or cones on their floors, and that these are very often in the center of these level spaces.

The radiating bands or streaks are beginning to appear.

In the Mare Imbrium, near the western end of the Alps, next north or Arispillus, is Cassini, of which the encircling cone appears to have been partly melted down by the lava of the mare so that it shows as a faint ridge with a distinct central crater.

(*a*) In this memoir all the features of the moon commonly termed "volcanoes," etc., are designated by the generic term "vulcanoid."

that many of these myths have not only become fixed in the minds of uninstructed people, but they have had a remarkable influence upon modern astronomers, limiting their capacity to interpret what their instruments clearly reveal to them. At every stage in the advance of selenography, we note the curious persistency of the endeavor not only to interpret the lunar features by the terrestrial, but to warp the observed facts into accord with those seen on the earth. There is perhaps no better instance of the extent to which prepossessions and prejudices may affect the judgment of the most conscientious observer, blinding him to evident truth, than the history of lunar inquiries afford.

The story of the physical conditions of the moon had best be begun by noting that the relation of our satellite to a larger sphere is not exceptional, but the most characteristic of all the relations of one stellar body to another. Of the planets in the solar system, all save the two nearest the sun, Mercury and Venus, have one or more smaller spheres circling about them. The relation of the sun to the several planets in a larger way repeats this plan of grouping lesser about greater orbs.

It is generally believed by astronomers that the celestial spheres have been formed by a process of condensation, due to gravitation, of matter which was originally widely diffused; that our solar system, before it was organized into the sun and lesser bodies, was in the form of a diffused nebulous mass of spheroidal form which extended beyond the orbit of the outermost planet. As this matter gathered toward the center, the material now in each of the planets and its satellites parted from the parent body, probably at first in the form of a nebulous ring, or spiral, which in time broke and gathered into a spheriodal mass. In that detached portion of the parent nebula the process of concentration was repeated, with the result that satellites, or, as we may term them, secondary planets were formed substantially as the greater spheres were set off from the sun. There are many questions and doubts concerning the details of this nebular theory, but that the evolution of our solar system, and probably of all stellar systems, took place in substantially the manner indicated appears to be eminently probable; it is, indeed, fairly well established by what we know of the distant nebulæ and by the rings of Saturn, which apparently contain the material which normaly should have formed one or more of its satellites, but which for some unknown reason have remained unbroken.

It it not certain at just what stage in the concentration of a nebula, a planet or a satellite may be set off from the parent body; nor can the present distance of the satellite from the main sphere be assumed as that at which the parting took place. It is possible that the concentration of the parent body had gone so far that the diffused or nebulous stage of its materials had been passed by and the more advanced stage of igneous fluidity entered on. It is, however, more likely that in all cases the separation occurred while the particles of matter were divided as they are in a gas or vapor. As soon as the two spheres are separated from one another, and so long as they remain in any measure fluid, the difference in their gravitative attraction on the nearer and more remote part of their masses induces tides, and the effect of these tidal movements, as has been shown by Prof. George Darwin, is necessarily to impel the two bodies farther apart. It seems certain that before the earth and the moon became essentially rigid, as they now are, the effect of these tides in driving them apart must have been great enough to account for a considerable part of the interval which now separates them.

In the present condition of the moon it is a sphere having a computed diameter of 2,159.6 miles and its mean distance from the earth 238,818 miles. So far as has been determined the moon exhibits no trace of flattening at the poles, such as characterizes the earth, unless, as is possible, there are irregularities of figure on the unseen part of the sphere. It is essentially a globular in form.

The measure of density of the moon—i. e., the proportion of its weight to its bulk—is only about six-tenths that of the earth. While the earth's mean density is nearly 5.7 times that of water, that of the moon is about 3.5 times as great. Thus the total gravitative force of the lunar mass is to be reckoned as only about one eighty-first of that of our planet.

As the moon revolves on its polar axis but once in about a month, and at a rate that tends to keep the same part of its surface turned toward the earth, we should, but for the phenomenon of librations, see no more than one-half of its superficial area. Owing, however to this feature, which is due to certain complications of the moon's exceedingly varied movements, the satellite in effect sways in relation to the earth so that at certain times we see farther to the east and at others farther to the west of its center, and in the succession of these movements we are able

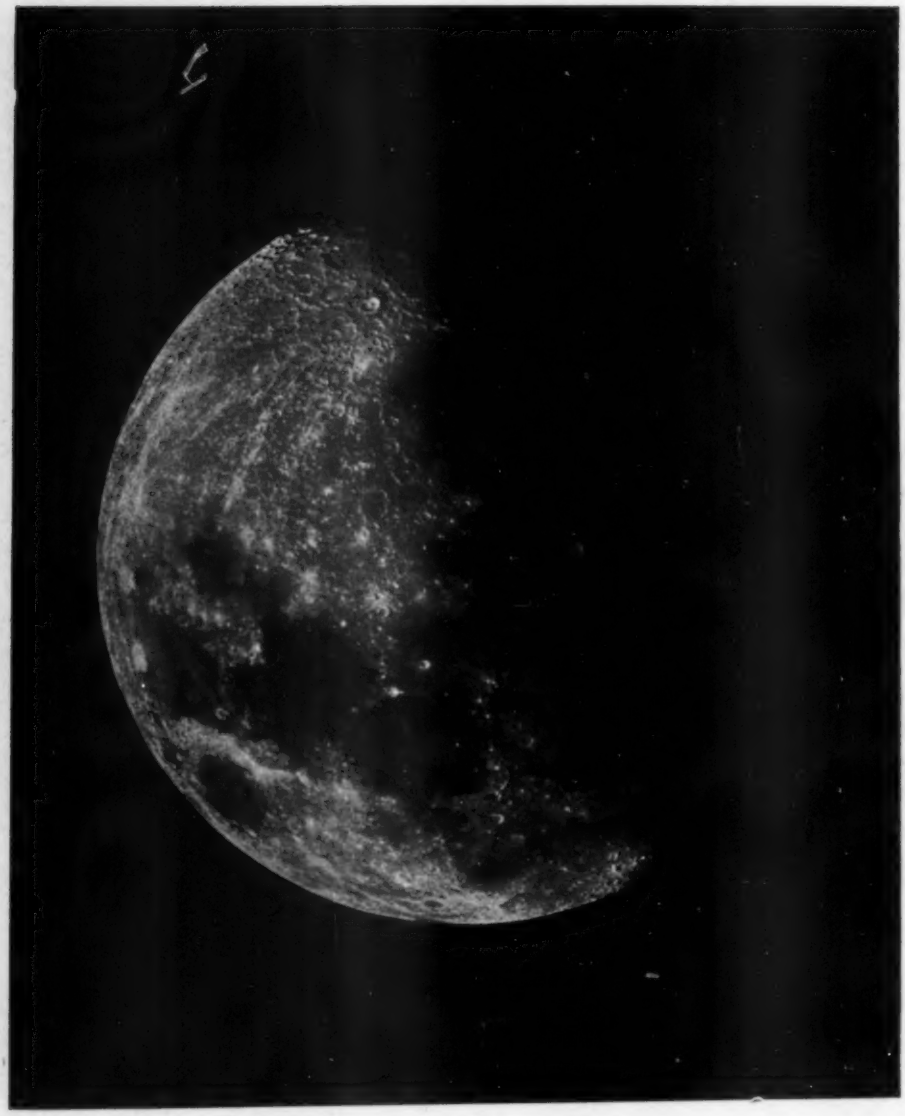

PLATE II.

Moon's age, 10 days 12 hours. Lick Observatory, 1890.

The most noteworthy changes as compared with Pl. I are the great advance in the development of the fields of very bright hue, and in the bands radiating from them. These are most evident in the system of Copernicus. The system of Tycho also begins to be evident. This vulcanoid may be identified as the deep large crater with a central cone near the border of the illuminated area. The general irregularity of these light bands is well shown in those about Copernicus. So, too, the fact that they are projections from an illuminated or lucent field about the vulcanoid.

The relative absence of large vulcanoids on the maria is noteworthy. Those which exist lie nearly, if not altogether, on fields of high ground which appear to have risen above the floors of the maria and so escaped melting.

The problematical crater Linne now appears as a small white patch near the middle of the eastern side of the M. Serenitatis.

to behold somewhat more than one-half the total area—in fact, about six-tenths of it. It is impossible to set forth in this writing the reasons for the librations of the moon, as the matter cannot be explained without giving in mathematical form a full account of the motion of our satellite, which is one of the most complicated of astronomical problems. (*a*)

As noted below, there is some accessible information going to show that even beyond the extreme field revealed by the librations the surface of the moon has the same character as that which is visible. * * * These several facts, taken together, make it eminently probable that the unseen four-tenths of the lunar surface in no essential way differs from that we observe. It is, indeed, altogether likely that we see every type of structure that exists on the moon, and that a view of its whole area would add nothing essentially new to our knowledge of the sphere.

Seen by persons of ordinarily good vision, even at a distance of about 240,000 miles, the moon reveals much of its surface shape, structure, and color; it is evident that the color varies greatly from very bright areas to those which are relatively dark, that the latter are somewhat less in total extent than the former, and that they are disposed in a general way across the northern hemisphere. (*b*) Persons of more than usually good vision may, under favorable conditions, see on the edge of the illuminated area the ragged line of the sunlight, which indicates that the surface is very irregular, the high points coming into the day before the lower are illuminated. Such persons at time of full moon can also note, though faintly, some of the bright bands, which radiating from certain crater-like pits, extend for great distances over the surface. So, too, they may see at the first stage of the new and the last of the old moon, the light from the sunlit earth slightly illuminating the dark part of the lunar sphere, or, as it is often termed, the old moon in the arms of the new.

With the best modern telescopes under the most suitable conditions of observation the moon is seen as it would be by the unaided eye if it were not more than 40 miles from the ob-

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(*a*) An excellent nonmathematical presentation of the question, which affords a sufficient idea of it, may be found in "The Moon," by Richard A. Proctor, pp. 117 et seq., D. Appleton & Co., New York, 1878.

(*b*) It is well to note the fact that in a celestial telescope objects are seen in reverse position, or "upside down." For convenience they are usually so depicted on maps and pictures of the moon; the north pole at the bottom, and the east where it is customary to place the west on terrestrial maps.

server. The conditions of this seeing are much more favorable than those under which we behold a range of terrestrial mountains at that distance, for the reason that the air, and especially the moisture, in our atmosphere hinders and confuses the light, and there is several times as much of this obstruction encountered in a distance of 40 miles along the earth's surface as there is in looking vertically upward.

Seen with the greater telescopes, the surface of the moon may reveal to able observers, in the rare moments of the best seeing, circular objects, such as pits, which are perhaps not more than 500 feet in diameter. Elevations of much less height may be detected by their shadows, which, because there is no trace of an atmosphere on the moon, are extraordinarily sharp, the line between the dark and light being as distinct as though drawn by a ruler. Elongate objects, such as rifts or crevices in the suface, because of their length, may be visible even when they are only a few score feet in width, for the same reason that while a black dot on a wall may not make any impression on the eye, a line no wider than the dot can be readily perceived. Owing to these conditions, the surface of the moon has revealed many of its features to us, perhaps about as well as we could discern them by the naked eye if the sphere were no more than 20 miles away.

Separated from all theories and perpossessions, the most important points which have been ascertained as to the condition of the moon's surface are as follows:

The surface differs from that of the earth in the fact that it lacks the envelopes of air and water. That there is no air is indicated by the feature above noted—that there is no diffusion of the sunlight, the shadows being absolutely black and with perfectly clean-cut edges. It is also shown by the fact that when a star is occulted or shut out by the disc of the moon it disappears suddenly without its light being displaced, as it would be by refraction if there were any sensible amount of air in the line of its rays. This evidence affords proof that if there is any air at all on the moon's surface it is probably less in amount than remains in the nearest approach to a vacuum we can produce by means of an air pump. Like proof of the airless nature of the moon is afforded by the spectroscope applied to the study of the light of an occulting star or that of the sun as it is becoming eclipsed by the moon. In fact, a great body of evidence goes to show that there is no air whatever on the lunar surface.

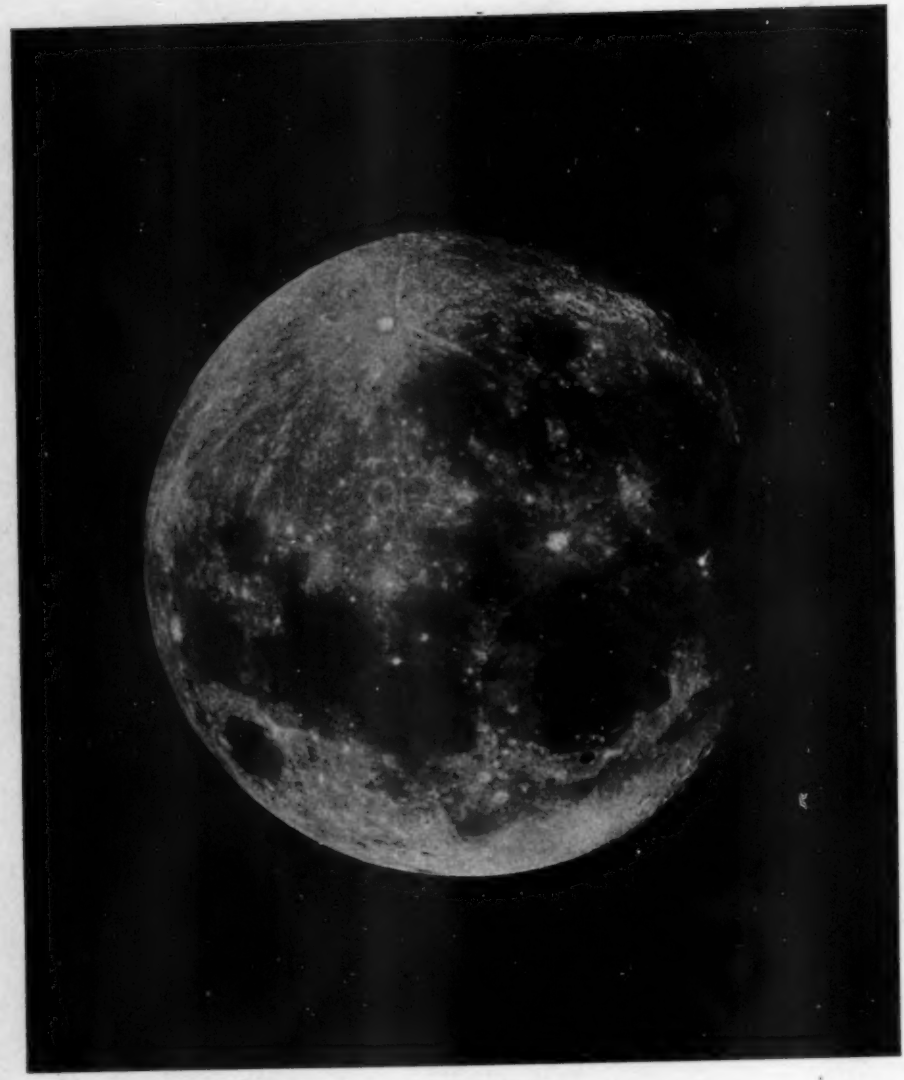


PLATE III.

Moon's age, 14 days 1 hour. July 19, 1891. Lick Observatory.

In this plate the moon is nearly full, the light being oblique enough to illuminate the crater walls on the eastern margin alone.

The maria are well shown nearly to the eastern margin. Separated by a belt of relatively high ground from the Oceanus Procellarum is the large vulcanoid Grimaldi. It has a small crater on its floor near its northern side. This vulcanoid has a floor nearly as dark as the seas. It will be noted that Plato has also a dark floor. On the margin of the Oceanus Procellarum, southwest of Grimaldi, is a crater Letronne rather indistinctly seen, the wall of which that faces the maria is, as in other instances, ruined apparently by the lava of the sea. Other like examples are shown in this neighborhood. On the shores of the M. Humorum there are three similar instances of crater walls broken down on the seaward side.

It should be noted that none of the maria distinctly attain the margin of the moon's surface. On the eastern lands the O. Procellarum comes near to the border of the moon, but high, rugged land is obscurely visible on the very edge. This is more clearly disclosed at certain stages of libration. On the southwest border some observers think there is a nearly level area crossing the border, but, as will be seen, the level land there has not the characteristic dark hue of the maria.

It will be observed that in this nearly vertical light, except Plato, the craters on the eastern margin only are distinctly visible. Those exceptions are due to the dark color of their floors. There are two or three craters near the south pole which, because they have rather dark bottoms, are faintly seen.

The evidence of lack of water at the present time on the surface of the moon appears to be as complete as that which shows the lack of an atmosphere. In the first place, there are evidently no seas or even lakes of discernible sizes. There are clearly no rivers. If such feature existed, the reflection of the sun from their surfaces would make them exceedingly conspicuous on the dark background of the moon, which for all its apparent brightness is really as dark as the more somber-hued rocks of the earth's surface when lit by the sun. Moreover, even were water present, without an atmosphere there could be no such circulation as takes place on the earth, upward to clouds and thence downward by the rain and streams to the ocean. Clouds can not exist unless there be an atmosphere in which they can float, and even if there be an air of exceeding tenuity on the moon, it is surely insufficient to support a trace of clouds. Some distinguished astronomers have thought to discern something floating of a cloud-like nature, but these observations, though exceedingly interesting, are not sufficiently verified to have much weight against the body of well-observed facts that show the moon to be essentially waterless.

The well-established absence of both air and water in any such quantities as is necessary to maintain organic life appears to exclude the possibility of there being any such life as that of plants and animals on the lunar surface. It may be stated that very few astronomers are now inclined to believe that the moon can possibly be the abode of living forms.

Being without an effective atmosphere, for the possible, but unproved remnant that may exist there would be quite ineffective, the moon lacks the defense against radiation of heat which the air affords the earth. Therefore in the long lunar night the outflow of heat must bring the temperature of the darkened part to near that of the celestial spaces, certainly to some hundred degrees below Fahrenheit zero. Even in the long day this lack of air and consequent easy radiation must prevent any considerable warming of the surface. The temperature of the moon has been made the matter of numerous experiments. These, for various reasons, have not proved very effective. The most trustworthy, the series undertaken by S. P. Langley, indicate that at no time does the heat attain to that of melting ice.

(*To be continued.*)

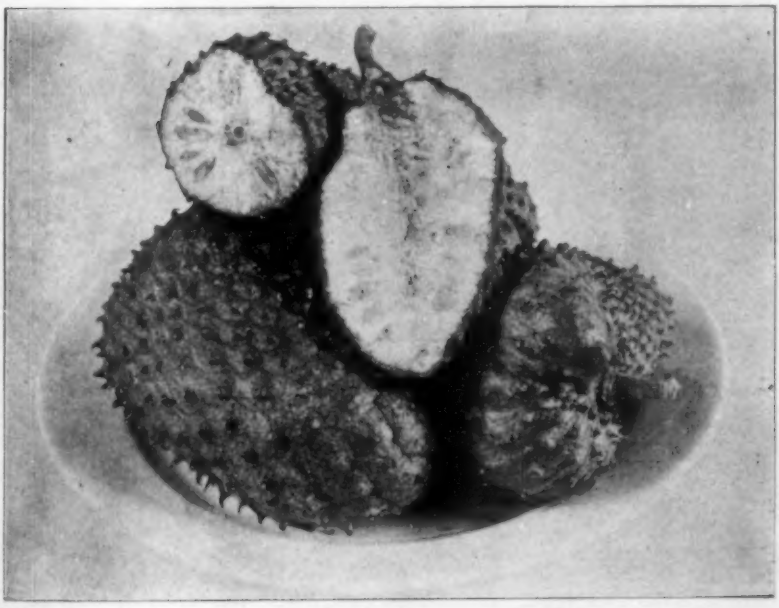
TROPICAL FRUITS.

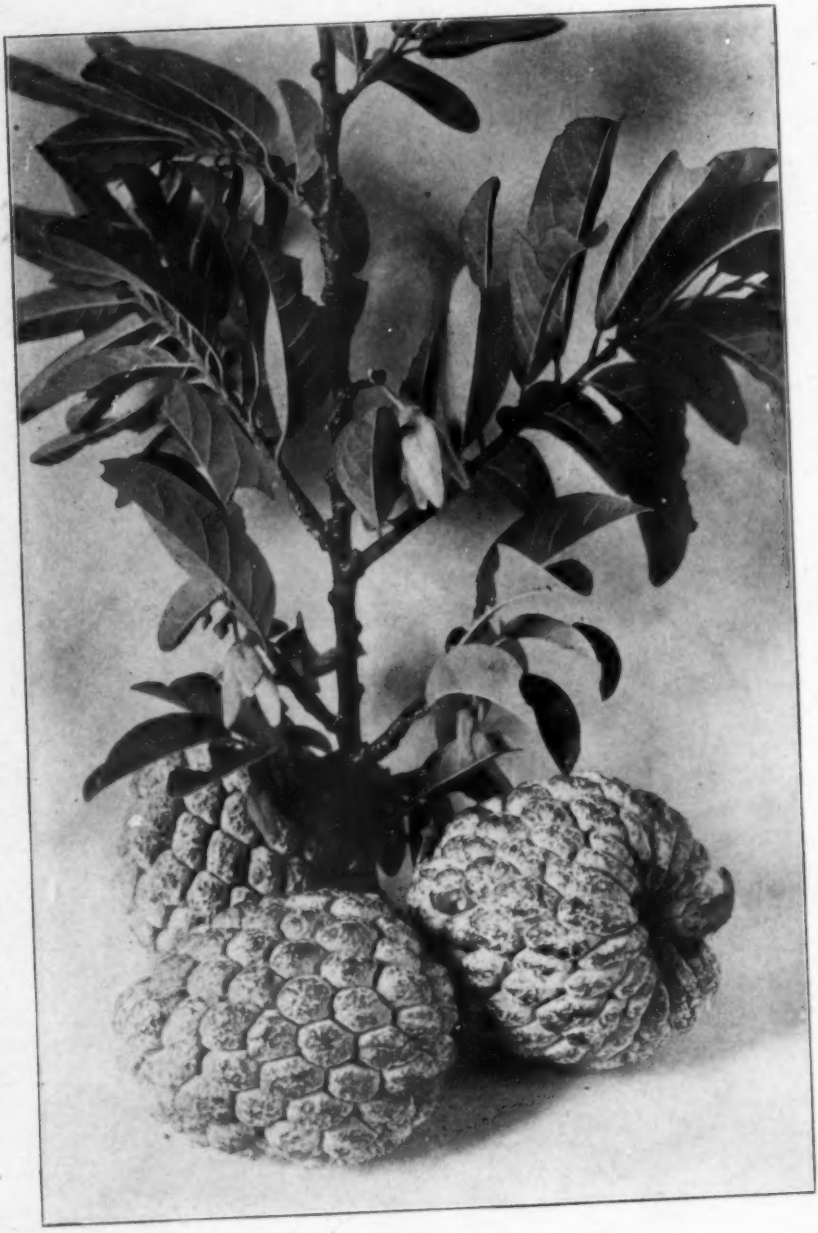
By Mel T. Cook,

Chief of the Division of Vegetable Pathology of the Cuban Agricultural Experimental Station.

Anonaceae. Custard Apple.

One of the most interesting things to the botanist in going from one country to another of entirely different climatic conditions is the comparative predominance of certain families of plants. One of the most striking illustrations of this kind is the family *Anonaceae*. This family is represented in the region covered by Gray's and Britton's manuals by the single genus and species *Asimina triloba*, Dunal, commonly known as the North American Papaw, while in the tropics it is represented by about 50 species. The *Asimina tribola* has a very wide distribution from the cold regions of Ontario to the warm regions of Florida and from New York westward to Nebraska, but the tropical species are unable to extend themselves into any such wide ranges of climatic conditions. Our single northern species has a very conspicuous rich purple flower with spreading petals,





with many stamens and with 3 to 15 carpels of which each is capable of maturing into a fruit about 3 or 4 inches in length. However, usually only three or four of these reach maturity. The tropical species usually have much less conspicuous flowers of the same general character as our northern species but the carpels unite to form a single compound and much larger fruit.

The most predominant genus of this family is *Anona* which as previously stated contains about 50 species, mostly in tropical America but a few are found in Asia and Africa. Both the family and the genera are imperfectly known both by the botanist and the horticulturist. However, the fruits are among the most important on the tropical market and no one can avoid being impressed with their peculiar appearance.

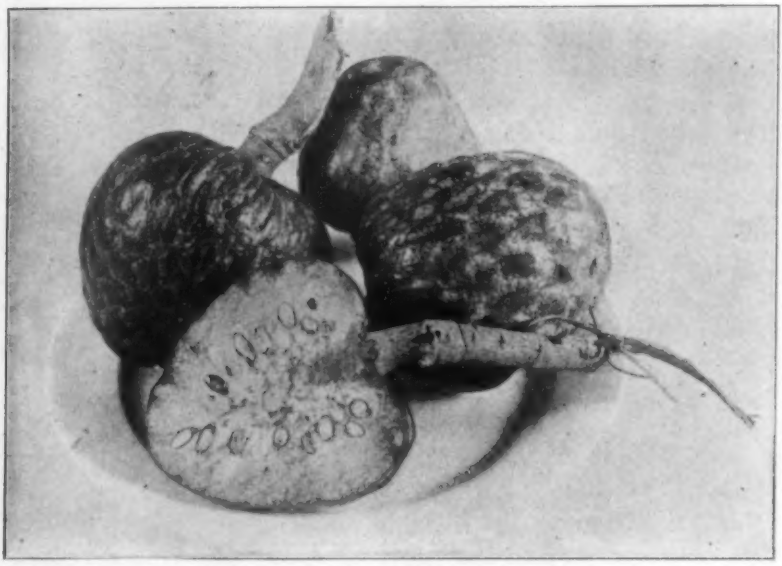
The largest and most conspicuous is the *Anona muricata*, Linn. commonly known as guanabana and sour-sop. These trees attain a height of 30 or 40 feet, are evergreen and have long eliptical, pointed leaves. The flowers are inconspicuous, yellowish or greenish on the outside and yellow or reddish on the inside. The fruit is quite large being 6 to 8 inches in length and weighing from 1 to 5 pounds. It is more or less oblong, blunt, dark green with a rough, spiny skin. The pulp is soft, white, juicy and with a slightly turpentine flavor. The seeds are arranged in rows in the pulp. It is very common on the market and is extensively used for making an excellent drink and as a flavoring for sherberts, ice creams, desserts, etc. It is a very palatable and pleasant food.

The *Anona cherimolia*, Mille., commonly known as cherimoja, cherimoyer, or Jamaica apple is a tree 15 or 20 feet in height and with flowers very similar to *A. muricata* except that they are less conspicuous. The fruit is about 3 or 4 inches in diameter but may be as much as 8 inches. It is more or less spherical, nearly smooth, brownish yellow, sometimes tinged with red. The pulp is soft and rich and very delicious. It is not used so extensively in ices, etc., as *A. muricata* but is much better as a raw fruit. It is very abundant on the markets in season.

The *Anona squamosa*, Linn. is a tree ranging from about 10 to 20 feet in height with flowers that are even less conspicuous than either of the preceding. The fruit is about 3 to 5 inches in diameter and yellowish green, each carpel forming a protuberance. It is the finest flavored of any of this class of tropical fruits

and in season can always be found in great abundance on the local markets.

All three of these fruits and also several other species have been introduced into southern California, Florida and southern Europe. If it were not for their very poor shipping qualities they would become very popular on our northern markets. This is one of the most important families of plants for future study both by the botanist and the horticulturist.



ARTIFICIAL RUBIES.

By Dr. H. G. Byers,

University of Washington, Seattle.

Considerable local interest has been stirred up in Seattle, Wash., over the publication of an article stating that rubies were being manufactured by the writer at a cost of ten cents each. This article has been copied widely and with various degrees of misrepresentation, so that it will perhaps interest readers of School Science and Mathematics to know the true state of the case.

It may be well to state at the outset that the main lines of

the newspaper tale are true and only in error as to details. One of these details is that the writer is one of the interested parties.

For some time M. Dorisey, a chemist connected with the chemical work of the city of Paris, has been experimenting with the hope of improving on the earlier efforts to make gems artificially. He was signally successful with the ruby, and was able to make pure gems of this variety from ordinary alum and oxide of chromium, long known to be the only constituents of the natural product.

He was induced to bring his project to this city because of the large market in this country for jewels and the 60 per cent. duty on the importation of the same. He located in Seattle with the help of a naturalized Frenchman of means and a Parisian cousin by the name of Richards.

Here, after setting up a rather elaborate plant, they were unable to manufacture clear jewels, because of spots and bubbles which appeared in them, in spite of their utmost efforts to correct the fault. They spent several thousand dollars on the project and were nearly ready to give it up in despair. At this juncture they consulted the chemists of the University of Washington, who were happily able to point out to them the fact that the local gas supply, being of carburetted water gas, and American oxygen were responsible for their troubles—the former by furnishing the black spots and the latter, since it contains from 10 to 20 per cent. nitrogen, for the gas bubbles. The French gentlemen then moved their plant to a point where they could procure coal gas, and by use of pure oxygen were then able to procure gems of such freedom from flaws, that in this respect only are they distinguishable from the natural stones.

Of course these investigators do not desire to make public their method of work, since such publication would interfere with the financial benefits which will undoubtedly accrue from their discovery. But there are some points which are of general interest to teachers of chemistry.

The method is to heat aluminium oxide and chromium oxide to such a temperature as will cause them to fuse and then slowly crystalize. In this manner the substance develops the beautiful color characteristic of the ruby; and yet, only occasionally is the crystalization so perfect as to show the faces of the crystals. The melted mixture apparently shows the same behavior characteristic of melted sulphur, viz., on higher heating it becomes more viscid

than just at melting point. Also the melted mass seems to occlude nitrogen in a greater degree than the oxides of iron and cadmium.

The items of cost as stated above arose perhaps from the fact that the raw materials are so inexpensive. The operation is, however, not so simple as would be inferred. The time required to make a stone of any considerable size is upwards of an hour and a half after all preparations are made. Even then, the adjustment of temperature is so delicate that many imperfect stones are made. These then must be cut and polished after the manner of the natural gem. There is, therefore, no immediate prospect of the market being flooded with a cheap imitation gem.

There is also added interest in the fact that in no sense may these jewels be considered as imitations. They are as hard, as beautiful in color, more free from flaws and of exactly the same composition as the natural article. It is curious to note, however, that since the name of the writer of this article was published as being connected with the manufacture of rubies, he has been the recipient of many letters from various persons who desire to palm off paste diamonds, yellow diamonds, and other imitation jewels as the real article, and who ask help to perpetrate their imposture. He was also the recipient of twenty cents from a Brooklyn gentleman, who desired a ruby, doubtless figuring that 100 per cent. profit was ample. He is not familiar with the western spirit.

COEFFICIENT OF LINEAR EXPANSION OF MERCURY.

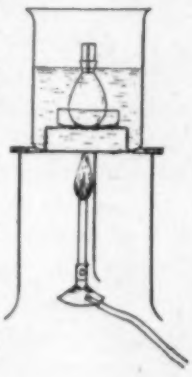
By Chas. H. Slater.

Wm. McKinley High School, St. Louis, Mo.

A new and very simple method of determining the coefficient of expansion of mercury has been used the past year in our laboratory with very satisfactory results. This method was suggested by *"The Weight Thermometer,"* Glazebrook and Shaw's Practical Physics page 259, also Deschanel's Natural Philosophy by Professor Everett, page 253.

A 25 c. c., or 50 c. c., specific gravity bottle was weighed empty and filled with mercury at the temperature of the room. The temperature was taken to the tenth of a degree with the thermometer in the earthenware bottle of mercury. The weighings were made to the tenth of a grain on a Harvard Platform bal-

ance. The bottle was then placed in a beaker of water as shown in the diagram, and the water boiled until the mercury ceased to flow from the capillary tube in the stopper. The temperature of the boiling water was noted, the bottle removed and again weighed. The bottle, while being heated, was placed in an evaporating dish to catch the expelled mercury. Both were placed on a wire gauze shelf to insure the mercury being heated only by the hot water.



From the data thus obtained the following were computed:

(a.) Weight of mercury remaining in the bottle.

(b.) Weight of mercury expelled by heating.

(c.) Change of temperature which caused the expansion.

(d.) Mercury expelled, i. e. expansion, per deg. C. (b. ÷ c.).

(e.) Expansion per deg. per gram (d. ÷ a.), i. e., the apparent coefficient of cubical expansion.

(f.) The true cubical expansion may be obtained by adding .000026, the coefficient of cubical expansion of glass, to the apparent coefficient of cubical expansion obtained.

The weight of the mercury (a.) at the temperature of the room would expand an amount represented by the weight of the mercury (b.) for the observed change of temperature (c.). Since weights are proportional to volumes, the coefficient of expansion may be computed, as above indicated, from the weights directly.

The average result of three large classes was .000182; correct value .000181. The two results farthest off were .000156 and .000206. Two-thirds of the results obtained had an error of 5 per cent. or less.

The accuracy possible by weighing a relatively large amount of mercury in the bottle instead of measuring the length of a small amount of mercury in a tube, and the ease of manipulation make the method well suited for use in the high school.

THE METRIC SYSTEM IN METRIC COUNTRIES.

By Dr. T. C. Mendenhall,

Formerly Pres't. of Worcester, Mass., Polytechnic Institute.

A few weeks ago I was walking on one of the perfectly constructed highways of Switzerland with America's most eminent astronomer (this is sufficient identification, except, possibly, among the astronomers themselves) and as we passed a "milestone" on which the distance from the frontier was shown in kilometers I said, "this reminds me that I have been asked to write something about the use of the metric system in Metric Countries." "And what is there to be said about the use of the Metric System in Metric Countries?" was the prompt and not very encouraging response. Perhaps, nothing;—and surely a very little would be enough were it not for the fact that some of those well meaning, but not well-informed people, who have set themselves against the use of the system in the United States, have declared that in Metric Countries the metric system is not used or, at least, that it is not in general use among the masses of the people. Having lived during the past four years in countries in which the metric system has long been the legal system of weights and measures, the results of my observations may be of some value. I do not suppose there is much difference of opinion as to France, where the system originated a little more than a century ago; the other countries, to which my observations during the past four years have been restricted, are Portugal, Switzerland and Italy, and mostly the latter. I have not always been in larger cities but have spent many months in country towns and small villages, especially in Switzerland. I have spent may hours among carpenters, blacksmiths, machinists, in public markets and elsewhere, where measuring and weighing are fundamental operations, having an undiminished fondness for killing time in that way. My conclusions may be summarized in the statement that among the masses of the people of these countries the metric system of weights and measures is as universally used as is the decimal system of currency among the people of the United States; surely not less so, and perhaps more so. One occasionally meets with a survival of antique measures, just as one not infrequently (in certain parts of the United States) encounters the "shilling" or "sixpence," so common during the first half of the last century. In Italy the most tenacious of the old units is the *libbra* or pound.

The word itself tells how deeply rooted it must be after centuries of use, and the real surprise is that it is not oftener met with. In value, however, it is most vascillating and uncertain for it has no legal recognition, and it varies "both in space and time." A half dozen different "authorities" will give as many different metric equivalents for it, ranging from a "mezzo-kilo" (500 grammes) to 320 grammes. In practice I have never seen it exceed a third of a kilo, (333 1-3 grammes) although several Italian dictionaries declare it to be the equivalent of 340 grammes. Oftener it is 320 and sometimes as low as 300 grammes. Of course, it is understood that there is no material representation of the *libbra;* every man who uses it fines its value in grammes to suit the transaction, striving to make it high in buying and low in selling. Naturally there are many honest dealers who wish to treat fairly the *libbra* and the very few who insist on using it, and the general custom among them is to value it at one-third of a kilogramme, prices *"per libbra"* being fixed on that basis. But in actual weighings the *libbra* is made to be only 330 grammes, "because we have no 3 gramme weight"—thus making a difference of about one per cent. against those buying by the *"libbra,"* even under the most favorable conditions and among the great majority of sellers (I have never seen it used except in small retail transactions) only 320 grammes are given, affording a profit of four per cent. at the expense of the lover of the old "pound." Among those whose consciences are more elastic the margin of gain in selling by the *libbra* is generally greater than this. With them it is a favorite, and they encourage its use, while the great majority of respectable dealers dislike it and avoid its use as far as possible, looking upon it as an uncertain and fluctuating standard of measure, having no "material representative." The kilogramme with its multiples and sub-multiples, together with the balances or "steel-yards,"with which it is used, are subject to frequent tests by the government authorities and the accuracy of those in use are altered by the stamp of the inspector. The *"libbra"* is an intangible ghost of a defunct unit.

I have been much interested to note a fondness for the use of this Italian pound where it might be least expected, namely, in the "English" and "Anglo-American" shops, much frequented by English and American travelers who imagine (and it is often imagination only) that they are getting better "goods" than can be found in the shops of the "natives." Of course, the metric

system is in use in these shops but an Englishman or an American is very likely to ask for "pounds" or "ounces" of things, and he often never discovers that at best he gets less than three-quarters of what he thinks he is getting. Indeed, selling by the "pound" to such people is a tidy bit of business and the shop-keepers cannot help liking it. I have no doubt that an American tourist spending a few weeks in Italy might go away with the impression that the "pound" is still a much used unit.

I was nearly three years in Italy before happening to meet with the *braccio* (arm) an antique unit of length. It was at the ,'Annual Fair" in a conntry town (whose Etruscan foundations were laid before Rome was thought of) and two old women were bargaining with a peddler for material for dresses. They knew by tradition and usages the number of *braccio* necessary for their garments and when the sale took place I observed that the *braccio* was 58 centimetres, the measurement being made directly off of the "meter-stick." As with the *libbra* the value of this unit varies in different localities and as far as I have observed it is very rarely used. A bright, young Italian girl who knew *il metro* very well, having studied weights and measures in the schools, was quite surprised when I asked her about the *braccio,* having never heard of it. In considering the very infrequent and really unimportant survival of these ancient units one should remember the burden of ignorance and illiteracy under which the "masses" have long existed in many European countries, happily now being lifted in some of them. But it is only since the House of Savoy began at Rome to rule over a united Italy (only a generation ago) that anything like the general education of the masses has received attention; and in the insular province of Portugal where I spent many months only about one-twentieth of the population could read and write.

A change in the system of weights and measures would be enormously facilitated by the wide spread intelligence of such as constitute the "masses" in the United States.

I have been greatly interested in observing the ease and accuracy with which the metre and its sub-divisions are used by mechanics of all sorts in laying out and executing their work. I gave some attention to this because of the oft repeated assertions of the opponents of metric reform that a decimal system did not lend itself to easy application and use in ordinary operations of measurement and their intimations that they had discovered

some mysterious adjustment of the human mind in virtue of which it works well only in halves, quarters, eighths, etc. The argument never amounted to much, especially in America, for it was so completely negatived by the decimal currency of which everyone is proud.

In Europe a much larger part of the "manufacturing" is still done (as the name implies) "by hand," than in America and in every village carpenters, cabinet-makers, carriage builders, etc., may be found, doing coarse and fine, cheap and costly work, as was the case in America fifty years ago.

I have spent pleasant hours in watching them and in occasional discussions of methods and implements. Cabinet-makers and carpenters carry the folded metre rule, as ours carry the "two-foot rule," the metre being usually folded in six parts and almost invariably graduated to millimetres throughout its length. I have never been able to detect the slighest difficulty in the use of the metre and its decimal sub-divisions because of either the nature or dimensions of these sub-divisions. When I showed a bright Swiss workman our own yard and explained to him its divisions into feet, inches, halves, quarters, eighths, sixteenths, etc., a look of wonder came upon his face, followed by a smile, just such as I have seen on the face of an American when he first saw an Englishman wrestling with a problem in interest, in pounds, shillings and pence, and which very distinctly expressed his astonishment that a great people, otherwise so intelligent and progressive could be so inexpressibly stupid as to cling to a system of mensuration so clumsy, so time wasting and so unscientific. He did not know how dearly we love our inch and our pound! !

Florence, Italy, October, 1905.

A CRITICISM ON "ARE ATOMS DIVISIBLE," FROM THE NOVEMBER, 1905, ISSUE.

In looking over your issue for November, 1905, my attention has been held by the article "Are Atoms Divisible", by Arthur A. Skeels of Cleveland, Ohio. Mr. Skeels seems, like a great many fairly well informed students of Physics, to misunderstand that ridiculously paradoxical statement which has been spread far and wide by some of our older text books on physics, namely, that "light waves are not only transverse but they vibrate in

every plane passing through the line of direction of the light ray." I quote from the author's paper. He evidently takes it that this vibration in every direction is understood to mean in every direction at once, and I do not wonder that he declares it to be inconceivable and that he proceeds to develop a new theory of his own to account for polarization.

The fact is that at least a million successive waves in a ray of ordinary light are exactly alike, are polarized, plane or elliptically polarized, and have vibrations of a definite simple kind. This train of similar waves then comes to an end on account of collison or other disturbance in the original molecule or molecules from which the light emanates, and a new and entirely unrelated train of waves follows, which in turn peters out, to be followed by another unrelated train and so on. The result is that during any perceptible interval of time an utterly confused horde of unrelated wave trains pass by in a beam of ordinary light, and no direction at right angles to the ray stands out as the dominant direction of vibration nor does any type of oscillation, elliptical, circular, or straight, stand out as the dominant type.

I believe that this brief statement will not be amiss in "SCHOOL SCIENCE AND MATHEMATICS" for I have made it a point to question, as a matter of curiosity, a great many persons and what surprises me is that very many accept the statement as to the vibration of a ray of light in every direction at once, and are not driven by the force of its absurdity to build up a new theory of polarization of their own.

W. S. FRANKLIN,
Lehigh University.

NOTE ON THE TERM LOGARITHM.

BY G. A. MILLER.
Stanford University.

This note is simply an appeal to banish the term logarithm from mathematical text-books, and to replace it by the term exponent. Instead of "Table of Logarithms" it is suggested to use "Table of Exponents." Instead of saying "Logarithm of a number" it is suggested that the phrase "Exponent for a number" be employed. As the terms exponent and logarithm have the same meaning it is suggested that the use of the latter be discontinued

and that a slight simplification in mathematical terminology be thus effected.

One of the main arguments against this change is that the gain may appear too trivial to deserve attention. It may be argued that a student will at once learn that common logarithms are exponents of 10 and that the use of the term logarithm cannot do much to retard his progress even if the term exponent would answer just as well. While this is probably true of the majority of students yet it is equally true that there are many exceptions. It is not uncommon for students to make mistakes in using logarithms, which they will correct themselves when they are told that logarithms are exponents of 10.

The use of the term exponent instead of logarithm would not only simplify the use of logarithmetic tables, but it would also emphasize the laws of exponents by practical applications. These general laws should be thoroughly mastered by every student of algebra and it seems unfortunate to miss a good opportunity to exhibit their utility. If different terms are employed the student is apt to expect a difference of meaning and he has a right to expect it.

The term logarithm came into use before the theory of exponents was well developed. In Napier's first system of logarithms no base is explicitly recognized. When it became known how naturally logarithms flow from the general law of exponents this term should have been shelved with the numerous other terms which are included in more general ones. At any rate, it seems worth while to consider whether it is not time to dismiss the term from our text-books during this epoch of unusual efforts toward more effective mathematical instruction.

FUTURE OF THE MISSISSIPPI RIVER—THE EFFECT THAT DESTRUCTION OF FORESTS WILL HAVE UPON ITS HEADWATERS.

By Chas Cristadoro.

The forests have been looked upon by the settler both as an enemy and a friend. Unless the land was cleared of trees, no crops could be raised and so with an ax and a saw he felled them. Yet they gave him the lumber for his house and kept his fireside

alight and warm during the long winter days and nights. So fared the giant black walnuts of Indiana in the early days when the massive logs were split and hewed into fence rails, those remaining in excess of the winter's need for fuel piled high afield and burned, as one would today, in clearing a field of so much brush.

The great Michigan forests of white pine, that nodded to the summer's breeze and swayed before the winter's blasts, appeared not many years ago as inexhaustible and limitless, yet they in time disappeared and vanished as snow upon the yet warm earth, before the ax and saw of the settler and lumberman. The extravagances of the early lumberman would make the lumber operators of the present day bankrupt were they to follow them.

As the millions of buffalo disappeared from the face of the earth so have gone the forests of white pine that stood in a continuous, unbroken chain for hundreds of miles.

The forests were made for man to use, says the practical lumberman. 'Tis true—but only in a measure. They were made for man to use and for the use of man. So were the rivers. The water sources, trace them as you may, will be found in the forest. There the spring gives forth its swelling flow that makes the brook, that makes the stream and finally the river as it flows toward and empties itself into the ocean.

Remove the water protecting trees and you interfere with the supply that the springs give forth. In other words, history the world around reveals the fact that with the timber removed from a river's headwaters, so has the death knell of the river been sounded. Examples of this kind can be shown in all countries.

We must have water, whether it comes from subterranean or surface rivers or flowing springs and rippling brooks; it matters not, water we must have, without it we cannot live. To secure this commodity of nature, the great cities spend millions of dollars to follow it to its source, store and lead it to the cities for consumption. The ancient Romans left us a lesson in aqueducts that has been a speaking example.

With the destruction of the timber along the water courses, floods and drouths have followed. Many localities once blessed with abundant flowing water are to-day, at times, through drouth, absolutely deprived of it, because of the denudation of the timber on the sources of the river. Each State has suffered from the

encroachment upon its lumber forests and, in some cases, before it was too late, the legislature has stepped in to save the timber.

When Michigan was being rapidly divested of her great white pine forests, Wisconsin was being entered by the lumbermen as a fresh field for lumber exploitation. Minnesota's pine giants were yet untouched. But the day came when the lumbermen cleared Michigan and were swarming like bees into the pine of Wisconsin and then Minnesota's turn came. And now the end of Minnesota's timber is in sight, so much so, that those who have made millions through and by means of her pine forests are to-day investing them in the great fir, spruce and redwood lands on the Pacific Coast. The days of the white pine trees are numbered in Minnesota. And during these years one spot in the State of Minnesota has been kept sacred from the ax. It was the Chippewa Reservation covering 800,000 acres, 200,000 of which is water. From this spot the strong arm of the government held back the lumbermen. The pines were sacred and under their branches the Indians lived undisturbed.

Many were the covetous eyes cast upon this reservation as the pines beyond its borders became fewer and fewer. Many were the efforts to secure this land from the control of the Indians. Treaties were made and, as has been the case with all Indian treaties from the days of Columbus to date, broken.

Then an argument was put forth that the timber in many cases was dead and that windfalls were frequent and that such could be saved and the money given to the Indians, could the trees be cut and removed from the reservation. It was called "dead and down timber," the very name of which is so tainted with fraud, perjury and downright theft that it stinks in the nostrils of every man acquainted with its significance. But a law permitting the removal of the "dead and down timber" went through Congress, and it was stated that for every actually "dead and down" tree a thousand thrifty, growing white pine giants were laid low and removed. It grew into such a nauseating, scandalous steal that the very man who fathered the "dead and down" bill, although I will say his intentions were good, was ashamed of its workings.

An effort was successfully made through the Secretary of the Interior to stop this outrage. Then an attempt was made to have the Government protect and shield for the people, this beautiful sylvan paradise, for all time, like the Yellowstone Park, for

never did a person visit this region, but that he returned with but one wish and one sentiment, can it not be preserved for all time for the people?

Here is practically the headwaters of the Mississippi, although the river actually springs from Lake Itasca. But these are its headwaters for here are the three great lakes of Leech, Cass and Winnebigosh, with seventy smaller lakes, the infant Mississippi connecting them all and meandering among them, making one great checker board of stream and lake.

The picture of the giant pines growing even to the water's edge, the wild rice mantling the crystal stream and the phantom-like passing of an Indian laden birch bark canoe made a picture that took one back to the days of Fenimore Cooper and made one glad that such a primitive spot yet rested upon God's footstool. With the growth of pine properly cared for under forestry rules this great watershed gave promise of feeding the Father of Waters for all time.

But two billion feet of standing timber was a great temptation to the lumberman and townsite operators, anxious to "skim the cream," and therefore the Morris Bill was passed and this great woodland paradise was thrown open to settlement and the will and wishes of the lumberman and the land speculator.

And now what has happened? The Panama Canal is to be. The whole Mississippi Valley wakes up and looks at the map and cries "We must now see that our river is deepened. We must profit by the Panama Canal and we must have a navigable highway and Congress must help us." Suppose it does take $50,000,000 to deepen the channels of this great river, they say it's worth it and they want it done. And while the knocking for this $50,000,000 appropriation is heard upon the doors of Congress, so too is heard at the same time at this river's headwaters the chug, chug, chug of the keen axe, the rasping of the saw and the crash of the falling giants as they topple daily to the earth in thousands.

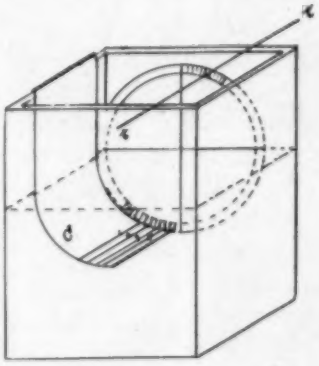
The headwaters are being cut out, the very headwaters whence this mighty river is draining her supplies, and the people of the Mississippi Valley, oblivvious to what is going on, are beseeching Congress to deepen the river. Cut out and dry up the river's headwaters first and then spend fifty million, yes, a hundred million dollars fruitlessly, to deepen the channel!—*Forestry and Irrigation.*

APPARATUS FOR DETERMINING THE REFRACTION OF WATER.

By P. G. Agnew,
Pontiac, Mich.

It seems to be always allowable to suggest another modification of the usual method of determining the index of refraction of water by means of the conventional rectangular glass jar.

In the form here shown in oblique projection, (scale 1-3), the protractor is pasted on the outside of the jar, as usual, but facing inward. Transverse lines drawn on a sheet of white celluloid *C* serve as a source of light. Angles in the water are read by sighting along three lines at the protractor. Angles in the air are read by stretching a fine black thread *tt* across the top of the jar and so placing the threadand the eye of the observer that one of the lines, the center of the protractor, the thread, and the eye all lie in the same plane. The intersection of the plane with protractor gives the angle.



The celluloid strip is shaped by placing in hot water, bending to the desired form, and allowing to cool. Better definition of the center of the protractor is secured by pricking a pin-hole through the paper and placing that side of the jar toward the window by which the room is lighted.

Careful manipulation will bring the maximum error of a single determination to less than 2 per cent., provided only angles of more than 10 degrees are used.

ON SCIENCE TEACHING. IV.

By C. R. Mann.
Ryerson Laboratory, University of Chicago.

In the preceding articles some of the general ideas which bear directly on science teaching have been considered. The remaining ones will be devoted to more practical questions. We shall direct our attention to the problem of determining how the science teacher may best prepare himself to carry out the principles out-

lined; for the question, "What can I do to strengthen my teaching and make it more vital?" must have occurred frequently to every earnest instructor.

I shall not attempt to give a categorical or ex cathedra answer to this question. Nor do I claim to be able to indicate more than the outlines of the solution of the problem for all the various branches of science. All that I can do is to call attention to some of the ideas that have helped me in making my instruction in physics more real and more interesting to my own students. I cannot but think, however, that these same ideas will be of assistance to those who are teaching sciences other than physics, if they will ponder them carefully, and apply them with such modifications as may in each special case be necessary.

The ideas to which attention will be directed are not new. They have been repeatedly emphasized and expounded by those who have written concerning the problem before us. I shall therefore indicate their sources and urge all teachers to study them in greater detail in the books in which they are set forth. The great and crying need of science teaching seems to be the practical application of these principles to actual instruction, and this each teacher must do for himself.

Perhaps the most important thing for every teacher to realize is the fact that no one can do good and really hearty work unless he has a clear and definite conception of the purpose or end towards which he is striving. Exactly what are you trying to accomplish by your labors? This is the great and central problem; for when a conscious aim has been developed in a teacher's mind, the battle is three-quarters won.

It is further clear that the quality of the purpose determines the quality of the instruction; for is not this a universal principle? Does not a conscious purpose precede every conscious act? And is not the quality of the act determined finally by the purpose that fathered it and whose expression the act is? Hence the conscious purpose of our instruction is the vital thing, and we should spare no pains or labor until we are sure that we have not only acquired one, but also assured ourselves that it is the best that can be acquired.

This purpose is also clearly a personal possession, which must inevitably be different, at any rate in detail, for each individual. For his teaching is part and parcel of the teacher's acts, and hence his purpose in it must bear the stamp of his individuality

perhaps even more markedly than do the purposes that impel him to his other actions. And if the purposes that lead a number of people to each do the same act are all different, as they surely are, we must recognize at once that it is not possible to state the purpose of science teaching in a way that will appeal to all science teachers as the complete and only one. Furthermore, every one who appreciates this fact has thereby laid on him the responsibility of forming his own purpose. He should also realize that in this matter his responsibility is great, since the results of his teaching depend on it; and these results may mean inspiration or despair, quickened thought and imagination or deadened aspiration, a joyful and optimistic or a querrulous and pessimistic attitude toward life for those growing personalities entrusted to his care.

When a teacher once comes to realize the importance and the potency for good or evil of his teaching purpose, he surely will not rest until he has made a careful and thorough study of the present bearings of his particular subject in the world of thought, of its relations to other subjects, of its development in the life of his race, and of the present day attitude toward it. All this may necessitate much reading, and even more thinking on the part of the teacher; but if he is in earnest, the result will justify the labor and his reward will be great. He will find that such study as this bears a far richer harvest than the continued picking away in the deep dark mine of his specialty; for it brings him into touch with the rest of the world and leads him out where he can feel the throbbing warmth of the life about him. He will be cured of that mental rheumatism which is so powerfully depicted by Goethe in his *Faust,* and which makes him cold, cynical, and out of touch with his time.

The science teacher is particularly qualified to form such a living teaching purpose, because his training tells him how to go about it. He is familiar with the scientific method of thought, and so knows where to begin. His first step should be to inform himself as to what has already been done in this matter. What ideas of the purpose of science teaching have been advanced?

In *The Teaching of Physics in the Secondary Schools,* by Prof. E. H. Hall, we read, in the chapter on Physics in the Various Kinds of Secondary Schools (page 327): "On the subject of this chapter we have something approaching the authority of official utterance in the various publications made by the Na-

tional Educational Association during the past ten or twelve years." Then follows portions of a report of a committee on College Entrance Requirements in Physics which was presented in 1899. After outlining the essentials of the course proposed, the purpose of the work is stated as follows: "To the end that the pupil may gain not merely empirical knowledge, but, so far as this may be practicable, a comprehensive and connected view of the most important facts and laws in elementary physics." This then is the official utterance of the National Educational Association—the aim of teaching physics is to give the student as far as may be practicable a comprehensive and connected view of facts and laws.

That there are, however, those who question the finality of this official and authoritative utterance, we are told on page 337 of the same book. For it seems that a course in secondary physics has been organized in Brookline, Mass., whose purposes are: "1. To develop in the pupil the habit of steady, persistent, logical thinking; 2. To render him fairly intelligent in reference to his scientific environment; 3. To beget a sense of power in his own ability to appreciate scientific truth and to draw legitimate conclusions from simple data; 4. To teach him to apply the elements of algebra and geometry to the problems of daily life; and finally, 5. To arouse in him a deep sense of appreciation of all that modern science has done and is still doing for the comfort and convenience of the race."

Another point of view is taken by Karl Pearson in his *Grammar of Science*. This author believes that the claims of science for public support and a place in the school curriculum are four. The first is: "That science, as training the mind to an exact and impartial anlysis of facts, is an education specially fitted to promote sound citizenship." The second is that science increases our knowledge and understanding of the laws of Nature and society. Since method and knowledge are both essential to sound judgment in civil and social life, science is practically valuable in proportion as it furnishes these two elements in education. But in the third place, "Pure science has a further strong claim on us on account of the exercise it gives to the imaginative faculties and the gratification it provides for the æthetic judgment." And finally, "Its last great claim to our support is the capacity it has for satisfying our cravings for a brief description of the history of the world." The science teacher in search of his pur-

pose should not fail to read carefully at least the first few chapters of this *Grammar of Science.*

One of the best and most useful monographs on the subject is the little book by Mrs. M. E. Boole, entitled, *The Preparation of the Child for Science.* This book pleads with teachers to recognize the fundamental distinction between the ways of teaching science and those of teaching languages. It is a simple yet powerful argument for guarding the freedom of the child's individual judgment. Thus we read (page 22): "In classical learning it is eminently desirable to secure that the right impression shall be made from the first; that the the wrong impressions shall have as little time as possible to deepen themselves. We wish the child's eye and ear to become accustomed from the first to welcome the right and reject the wrong; we do not wish him to gain any habit of tolerating wrong impressions. If the child uses the nominative where he should use the accusative, and is not at once corrected, that is so much to the bad for his future progress; if he can be got not to be able to remember a time when he used the word wrongly, that is so much to the good. But in science there are, there can be, no absolutely right impressions; our minds are not big enough to grasp any natural fact as a whole; everything depends on drawing right conclusions from combinations of impressions, each of which is in itself inadequate and partially misleading; and if the pupil is to be got into scientific methods, that is what he must be trained to do." And on page 26, "What science does claim is, that no child shall be told anything about the motion of the earth till he has observed many sunrises and sunsets; till a clear sense-impression of the earth standing still and the sun moving has become organic within him. This registering of a 'wrong' impression is what in science we have to secure; while in the classics we should try to prevent it." Further, page 32, "But when a child has formed for himself a clear, undisturbed impression of the earth's unmovableness and the apparent motion of the sun, and then has read that astronomers believe the earth goes round and the sun does not go round the earth, if he then puts together in his mind the two apparently conflicting statements and lets them combine to create in him an impression which shall embrace both, then the sacrosanct scientific act has taken place in his mind. He has really done a bit of true science work."

This book is full of excellent ideas. Note also the following:

"Scientific education is not arrived at, and never can be arrived at, by young people being crammed at school and college with ready-made knowledge; and left to find out, after adult life and its duties have come on them, that they are still very ignorant of how to learn what they now need to know. Nor do they mend the matter then, by turning in disgusted recoil from the sham knowledge with which others crammed them, to cram themselves from books, with ready-made theories about the limits of the knowable and the philosophy of the unknowable. Scientific culture is the result of a steady, life-long habit of friendly and intimate, though reverent, intercourse with the Eternally Infinite Unknown."

Still another valuable book on this subject is *Education and the Larger Life,* by C. H. Henderson. We are here told that the development of human power should be the aim of all education. We are reminded that great mental ability or a well-stocked memory does not atone for physical weakness. The physicist reads with interest his remarks about our proneness to overlook physical defects in our pride over our scientific achievements. Thus: "Our eyes are near-sighted and astigmatic and we can hardly recognize our friends across the street; yet with our telescopes we can study the details of the moon. Our voices are weak and squeaky; yet with the long-distance telephone we can shout the prices of corn from New York to Chicago. Even Stentor could not do that. The average man is short-winded and weak-kneed; yet the trolley car rushes him to his office, and the limited whisks him across the country." Every teacher will find much in this book to inspire and strengthen his work.

This subject is well nigh inexhaustible. There are many, many other works that bear in whole or in part on it. Three others should without fail be studied before forming a final teaching purpose. These are Froebel's *Education of Man,* Miss Blow's *Symbolic Education,* and Hanus's *Eduactional Aims and Educational Values.* All of these are so well known that they need no introduction.

The preface of almost every text-book on your subject contains some ideas bearing on this problem. Further, membership in an association of science teachers where such topics are discussed cannot but help in building up a strong teaching purpose. But the teacher must not forget that it is his privilege and his duty to form his own purpose for himself. A ready-made, stock-

in-trade purpose is a blind that does more harm than good. Unless his purpose bears the stamp of the teacher's individuality, it is not vital; he will not work enthusiastically toward its attainment. This finding of one's individual purpose may require considerable time, for it is impossible to reach the goal without much contemplation and careful thinking. Yet it is not time to realize that without deep thought the possibility of a comprehensive science does not exist; a wholly practical knowledge, directed entirely to facts and industrial pursuits, is, after all, of exceedingly little importance in the development of the human soul—a development which it is the teacher's highest duty to foster and encourage?

THE DEFINITION OF A LIMIT.

By Ernest B. Lytle,
University of Illinois.

In the article entitled "Limits in Geometric Forms," in the October number of School Science and Mathematics, the author is unfortunate in selecting a bad definition of a limit and consequently fails to see that the limit of a variable is an intrinsic property of the variable. Further, he magnifies the importance of the limit of a geometric form when the elementary teacher should be chiefly concerned about limits of variable numbers. We believe such an article to be of little benefit, if not actually injurious, to the teacher of elementary mathematics for whom it was written, because its wrong point of view leads to hazy notions of limits.

The definition of the limit of a variable number is of fundamental importance in the calculus, the theory of infinite series and the theory of functions. So important a definition should be clear, definite and simple as possible. In this article we purpose to show why the definition of a limit given in the above mentioned article is bad and to give a better definition; to show that the limit of a variable number depends only upon the definition of the variable and is an intrinsic property of the variable, and to show why the limits of geometric forms are of little importance, if not dangerous, to the teacher of elementary mathematics.

The definition of the above mentioned article is as follows: "A limit is that constant value (or form) which a variable value (or form) approaches indefinitely near but never quite reaches."

While we realize this to be a very common definition in the older texts still we believe it is narrow and otherwise bad.

This definition makes two demands; first, *that the difference between the limit and the variable may be made less than any assignable number,* and second, *that this difference cannot become zero.* The second demand narrows the definition by excluding the cases where the variable attains its limit. It is important that the definition of a limit include the many cases of variables which attain their limits.

For example, consider the limit of y as x approaches 3 when y is defined by the relations

$$y = f(x) = \frac{x^2 - 3x}{x - 3}, f(3) = 3.$$

It should be noted that without the second relation y would not be defined for $x = 3$ since, by substituting $x = 3$, we get for y the indeterminate form $\frac{0}{0}$. Evaluating the limit we get

$$\underset{x \doteq 3}{\text{limit}}\ y = \underset{x \doteq 3}{\text{limit}}\ \frac{x^2 - 3x}{x - 3} = \underset{x \doteq 3}{\text{limit}}\ x = 3.$$

This we see is a case where the variable y attains its limit, that is, y passes through a sequence of values which includes the limiting value 3.

This may be made clearer by plotting. Putting the equation

$$y = \frac{x^2 - 3x}{x - 3}$$

in the form

$$(y - x)\ (x - 3) = 0$$

we see that its graph consists of the two straight lines A B and C D. The perpendiculars dropped from the points of the line A B to the x — axis represent the values of y. Since we defined y so as to include the value 3, we see that as x varies from 0 to 3, y also varies from 0 to and including 3. It should be noticed that not the slightest approximation of any kind has been made. Here is a case where the difference between the variable and the limit becomes zero and is therefore excluded by the second demand of the above definition. This example is only a type

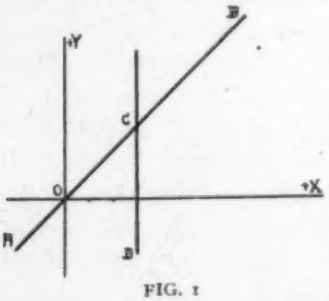


FIG. 1

of continuous functions, and the above definition is narrow because it excludes many limits of continuous functions.

When the limit of a function at a point exists and is equal to the value of the function at that point, then we say the function is continuous at the point in question; that is, the variable function, which is continuous at every point, passes continuously without jumps from one value to another. The definition quoted above of a limit is inconsistent with our idea of continuity since it will not permit a variable to attain its limit.

It is essential to clear thinking in limits that a careful distinction be made between *the limit at a point* and *the value at a point*. The value at a point depends only upon the definition of the variable, while the limit at a point is independent of the value at the point and depends only upon the values of the variable in the neighborhood of the point in question. For example consider the variables y and z as defined by the following relations:

(1) $y=f(x)=1$.

(2) $z=F(x)=1$ for x not zero, $F(0)=0$.

In both cases the limit as x approaches 0 is 1, while $y = 1$ for $x = 0$ and is therefore continuous, but $z = 0$ for $x = 0$ and is discontinuous; y attains its limit and z does not attain its limit. A limit does not depend upon the value at the point in question.

The definition of a limit should provide that the difference between the variable and its limit not only become but must *remain* less than any assignable number. The above definition does not make this provision. Consider y as defined by the sequence

$$0, \frac{1}{2}, 0, \frac{2}{3}, 0, \frac{3}{4}, 0, \frac{4}{5}, \ldots\ldots\ldots\ldots$$

The two demands of the above definition are fulfilled, for we can find a particular y^1 for which $|1-y^1| -< 1$, when 1 is any positive number however small, yet there is no y^1 which will make $|1-y^1| = 0$. However, few modern mathematicians would agree that the limit of y as here defined is equal to 1 because y does dot remain near 1 as it approaches it.

Another similar example is the limit, as x approaches 0, of sin 1. The value of this function oscillates between +1 and $\sin \frac{1}{x}$. The value of this function oscilates between +1 and −1, and the oscillations become dense at the point $x = 0$. The value of the variable not only approaches 0 as x approaches 0 but

it actually becomes equal to zero an infinite number of times in the neighborhood of the point $x = 0$. Yet we would say that this limit does not exist because as the variable approaches 0 it does not remain near 0.

Having seen why the above quoted definition is bad, let us proceed to discuss a better definition of a limit.

"When the law of change of a variable is such that its successive values approach nearer and nearer to a certain fixed number so that the difference between the latter and the variable can become and remain smaller than any assigned number, then the fixed number is called the limit of the variable in question." (Page 347, McMahon's Elementary Plane Geometry.) The equivalent of this definition may be found in Fine's College Algebra, Osborn's Calculus, Townsend and Goodenough's Calculus and Harkness and Morley's Introduction to Function Theory, and is being used by many mathematicians to-day.

This second definition of a limit also makes two demands; first, that for some value of the variable the absolute difference between the variable and the limiting value can be made smaller than any positive number which you may select, and second, that this difference between the variable and the limit *remain* less than the number you select. Since nothing is said about the variable never reaching its limit this definition includes cases of variables which attain their limits, as well as cases of variables which do not attain their limits. This definition is therefore consistent with our ideas of continuity. It should be noted that the question whether or not a variable attains its limit does not in the least affect the question of the existence of a limit. The second demand of this definition excludes such cases, as the two examples given above, where the variable approaches nearer and nearer a fixed value but does not remain near it.

This second definition has the advantage of permitting a simple geometric interpretation. Suppose we are given

$$\text{limit } y = \lim_{x \doteq a} f(x) = A.$$

The definition therefore requires that

$$|f(m) - A| < i \text{ and } |m| < n.$$

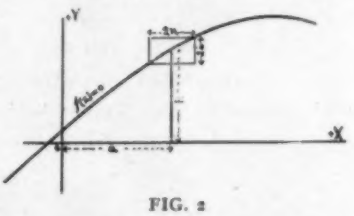


FIG. 2

Plotting we get the accompanying figure. That is, for any positive number i, however small, which you may select it is always

possible, when the limit exists, to find another number n, such that the graph always keeps within a rectangle, 2i by 2n in dimensions, constructed about the critical point on the graph. Such a geometric interpretation greatly assists in giving a clear idea of a limit.

Further, this definition has the advantage of permitting an extension to include limits of functions of two and three variables with corresponding geometric interpretations.*

This point of view has still the further advantage of avoiding all ideas of approximations. The method of limits is not a method of making approximations but a method of rigid demonstration.

We might easily carry this notion of a limit into the modern theory of assemblages (*Mengenlehre*), but such an extension is not within the scope of this article.

In accepting the second definition of a limit, we see that the limit depends entirely upon the sequence of values through which the variable passes, that is, upon the definition of the variable. A limit then is affected only by some change in the definition of the variable. We cannot agree, therefore, with the conclusion of the above mentioned article in SCHOOL SCIENCE AND MATHEMATICS of October, namely, that "the limit is the limit of the process and not necessarily any intrinsic property of the variable.". Consider the author's example, " If the angle X is generated by the motion of the intersection A, it has no limit; but if generated by the movement of the intersection B, its limit is a right angle." The angle X as defined by the motion of the point A is not the same variable as the angle X as generated by the motion of the point B because in the two cases X does not pass through the same sequence of values. The author is wrong in giving this as an example of the *same* variable having a limit by one process and having no limit by a different process. Since the limit depends only upon the definition of the variable we do not believe, after

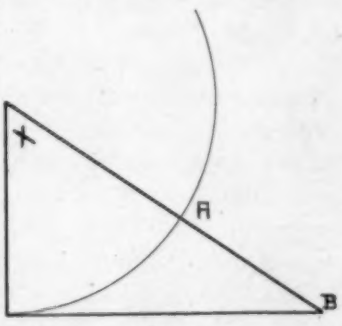


FIG. 3

*See "Double Limits" by E. B. Lytle, and "Functions of Three Variables" by Dr. H. L. Coar; theses in University of Illinois.

the variable has been defined, that a limit may be avoided by any specially selected process.

So far we have considered only limits of variable numbers. Limits of geometric forms are of very little importance to the teacher of elementary mathematics for whom the above mentioned article was written. At present we cannot recall a single branch of mathematics which makes any use of the limit of a geometric form. Further, most limits of geometric forms may be given analytic form and translated into limits of variable numbers. The analysis of numbers has been so much more rigorously and completely developed that most mathematicians use geometric forms simply as illustrations and put all demonstrations into the analytic form. We therefore, believe the author of the above mentioned article extremely unfortunate in laying such stress upon limits of geometric forms.

Consider some examples of limits of geometric forms. It is said the circle approaches a straight line as a limit as the radius becomes definite. Why not say the curvature of a circle $(\frac{1}{r})$ approaches the curvature of a straight line (o) as a limit as the radius becomes infinite. Again, it is often said the ellipse approaches a circle as its limit as the two axes becomes equal. Why not better say the limit of the ecentricity of an ellipse is the eccentricity of a circle as the two axes become equal. Instead of saying the limit of an inscribed polyglon, as the number of sides becomes infinite, is a circle, we prefer to define the length of a circle as the limit of the perimeter of an inscribed polyglon as the number of sides becomes infinite. Since all the limits met in elementary work may be thus stated in terms of limits of variable numbers the limit of a geometric form seems unnecessary to the teacher of elementary mathematics.

The great danger in reasoning by means of limits of geometric forms lies in the continual temptation to trust the eye for conclusions. Hilbert gives the following example of such eye fallacies:

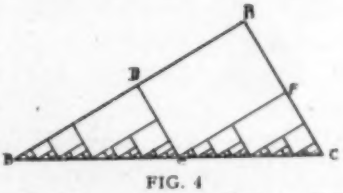


FIG. 4

In the triangle A B C draw D E and E F parallel respectively to A C and A B; then we have AB + BC = BD + DE + EF + FC. Repeat the above operation upon the triangles ADE and EFC. By repeating this operation an infinite number of times

the notches become smaller and smaller and the *form* of the broken line aproaches the *form* of the line AC as a limit. But the broken line is always equal to the sum of the sides AB and BC. Therefore the sum of two sides of a triangle is equal to the third side. By using geometric forms as illustrations and making our demonstrations analytic we avoid the danger of making such eye fallacies as this.

We have taken so many examples from the higher mathematics that the elementary teacher may be wondering what this article has for him. We do not believe in a full discussion of the subject of limits in elementary classes, but we do believe that elementary students should not be taught definitions which must be unlearned later. We should like to see elementary teachers omit the words "but never reaches" from every definition of a limit. Teachers should be very careful not to make the notion of a limit hazy and seem only an approximation by emphasizing in any way the non-esential fact that some variables never attain their limits. Moreover, they should avoid the danger of making eye fallacies in using limits of geometric forms, by stating all limits as limits of numbers. In elementary classes we would teach the following definition of a limit or its equivalent.

A variable x is said to approach m as a limit if the difference $m - x$ will ultimately become and remain numerically less than every positive number i that we may assign.

LOGICAL GEOMETRIES.

By Arthur Latham Baker Ph. D.

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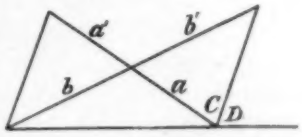
A recent article (School Science and Mathematics, Vol. V. p. 396) asserts that the logic of the majority of solid geometries is "vile", basing its assertion on the following statements: "To state without proof that the volume of the irregular solid lying between the lateral surfaces of the cylinder and inscribed prism * * * can be made less than any assignable quantity because its base can be made less than any assignable quantity, *is an assumption.* The only assumptions admissable in a logical geometry are axioms and postulates. The foregoing assumptions being capable of proof are not axiomatic."

The writer overlooks the truth expressed in the phrase which

is somewhat of a favorite with the public at the present time, that a geometry must be psychological as well as logical.

If it were necessary to go back to the foundations of geometry for every assertion or assumption the books would be so cumbered with metaphysical verbiage that only an expert could read them. They would cease utterly to be school books. In a school book we must make many obvious assumptions and trust somewhat to the intuition of the student to bridge the gap. This has been the custom ever since the time of Euclid.

In Bk. I, Prop. XVI, where b is prolonged its own length through the middle point of aa', the argument proceeds: but the

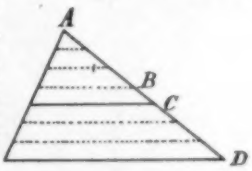


$$\angle (C+D) > \angle C,$$

and cites in support Ax. X, the whole is greater than any part.

This argument has passed muster for 2,000 years. I wonder if it is satisfactory to the writer of the article cited above. The tacit assumption that C is a part of C + D, that is, that the line between C and D falls inside the $< (C + D)$ is pure assumption of the boldest kind, dependent upon the looks of the diagram. It is not axiomatic and any attempt to support it by the citation of an axiom or even an antecedent theorem is futile. It has the additional vice that its so easily apparent validity is entirely dependent upon the *looks* of the diagram; a violation of one of the fundamental laws of geometric proof. It is placed among the earlier and foundational propositions of many books, as early as the 4th in some.

Similarly, in the proposition, a line parallel to the base of a triangle divides the sides proportionally. The regulation demonstration is to draw the parallels determined by the equal division on one side, assuming that these will divide the other side into the same number of equal parts and then count the parts. In this the assumption is made that: If B is between A and C, and C is between A and D, then B is between A and D. But this is the baldest kind of assumption. It is not axiomatic, being capable of proof. See Hilpert's Foundation of Geometry, Halstead's Rational Geometry, p. 254.



This kind of assumption runs all through the books. Why is not this logic as vile as that of the solid geometries. The books bris-

tle with such assumptions, but to supply the proofs would make the subject so appalling that few students would care to master the subject.

The assumption which Euclid makes in I. 16 was also equally arbitrarily made in I 6. To support it a new axiom must be established: If OA, OB, OC be three rays which meet a straight line not passing through O in A, B, C, and if B be between A and C then the $\angle AOB < \angle AOC$. In I. 1 there is no proof that the circles intersect. The assumption is not axiomatic, it can be proved.

It would not only not be worth the candle to supply such proofs to the student at the opening stage of the subject, but worse, it would be destructive of all interest and would practically thrust the subject out of the school curriculum.

The subject of geometry must be approached from the side of pedagogy as well as the side of logic, and many such points must be glossed over until the student sees and *feels* the need of a proof. The vast majority of students never feel the need, and it would be a pedagogical mistake to overload them with the to them unneeded and unnecessarily burdensome proofs of facts so very apparent and so unmistakably correct. It must be remembered that school geometries are not intended for the purpose of building up a logical chain of syllogisms from the metaphysical foundations, existence theorems and all, but are intended to boost the student along according to his apperceptive powers. Where we shall draw the line between rigorous proof and intuitive assumptions is an open question which must be decided by each generation and educational mass for itself. The rigor demanded in one place might be quite unnecessary under different circumstances and requirements, and not a bit more logical. An enthymeme is quite as logical as the more complete syllogism, and to style it vile because it is incomplete would be making a fetish of formalism. It is a mistake to suppose that logical accuracy is dependent upon full and formal completion of every possible syllogism. Many syllogisms need not be completed until the need for the completion is felt or until a doubt is raised for the completion to dispel.

Geometry in the schools cannot be a complete and rigorous chain of syllogisms and pedagogic at the same time. How far the rigor shall yield to the requirements of pedagogy is a question which no body of men can determine absolutely, but only

for their own time and circumstances. The logical development in a trade school, in the high school, in the college and in the university may be and generally are quite different. We must remember the object in view and the limited time which our crowded curricula give us for the attainment of that object. We must press forward with these things in mind, and not waste time and energy in exploring depths which the student is unconscious and unmindful of, nor in trying to drag a ponderous student through a capillary passage of the logical labyrinth. On the other hand, of course, we must not allow slovenly work. What is slovenly work must be decided by the circumstances. An obvious and evident and correct assumption whose proof would be difficult, tiresome and discouraging might not be slovenly, while an equally obvious assumption whose proof is easily made along the same lines as the other proofs of the course probably would be slovenly. Which side of the line the assumption as to limits in solid geometry lie, is still an open question. May be the next generation will close it. But the assumptions can hardly be relegated to the limbo of vileness yet; not so long as these other assumptions are accepted without question.

THE STUDY OF ACCELERATED MOTION BY MEANS OF THE INCLINED PLANE.

By N. F. Smith,

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The problem of uniformly accelerated motion is one which has long been a source of trouble both to students and teachers. This is due chiefly to the difficulty of providing suitable apparatus for its experimental study. The following adaptation of the

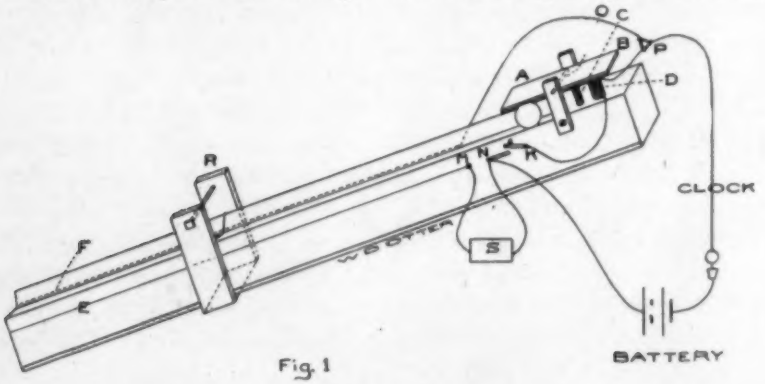


Fig. 1

inclined plane is one which can be used by any student and has given results of remarkable accuracy.

The plane consists of a straight piece of "two by four" about 16 or 18 ft. long having a **V**-shaped groove accurately planed in the upper edge. One end is supported in a suitable frame at an adjustable height. A steel ball 1 1-2 in. in diameter is held near the top of the plane by a wooden lever A B, (Fig. 1.) pivoted near the center at O and pressed up by the adjustable spring C. The end of the lever B. carries a soft iron armature and under it is mounted an electromagnet, D. By means of the contact Key K. an electric circuit can be closed through the magnet and the clock pendulum so that if the key is pressed the ball is released the next time the pendulum swings through the mercury contact.

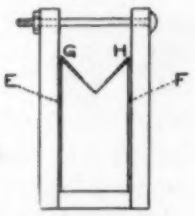


Fig. 2

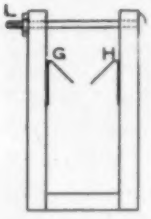


Fig. 3

Along each side of the plane is stretched a bare copper wire shown in cross section at E and F. (Fig. 2.) A clamp R, (Fig. 1,) shown separately in Fig. 3, bears on its inner faces two strips of thin copper G and H. This clamp slides along the plane with the copper strips fitting evenly in the **V** and when the set screw, L, is tightened the copper strips make contact with the bare wires E and F. These wires are connected one to the binding post M, the other to the wire from the clock at P. A sounder between M and N completes the outfit. It is evident from the connections that if the sliding clamp is so placed upon the plane that the ball rolls over it at the same instant that the pendulum swings through the mercury contact the circuit is closed and the sounder will click, but not otherwise. Hence by sliding the clamp along on the plane we may find exactly the distance which the ball rolls in 1 sec., 2 sec., etc. A scale may be ruled directly on the plane. If the clamp is set at the lowest point where a coincidence occurs the readings may be made to the upper edge of the copper strips. In this way settings may easily be made to an accuracy of half a centimeter and perhaps less. An interesting variation from the ordinary form of the

experiment is to determine the acceleration for two different heights of the plane when it can easily be shown theoretically if a_1 and a_2 represent the two accelerations and h_1 and h_2 the corresponding heights, that $\frac{a_1}{a_2} = \frac{h_1}{h_2}$

This formula has been verified with an error of less than one per cent . Another valuable modification is the use of the horizontal plane in connection with the inclined plane. Another piece of "two by four" precisely similar in arrangement to the first may be clamped horizontally in such a position that the ball rolls on to the horizontal plane at the end of 1 sec., 2 sec., or 3 sec., etc. The distance it travels in the next second can then be measured by the same means previously employed thus giving the velocity acquired in different lengths of time. If the farther end of the second plane is lowered below the level of the other, so that the ball will just continue to move if set in motion, the retarding effect of friction may be eliminated.

A considerable experience has convinced me that a student's understanding of a problem is vastly increased by actually verifying his theoretical equations by experiment. And nowhere is this more needed than in the problem of uniformly accelerated motion. To do this successfully the apparatus must be capable of yielding results of a fair degree of accuracy.

The apparatus described has proved so satisfactory for this purpose that it is recommended by the writer to other teachers.

Influence of Air on a Falling Body.—This may be very simply and clearly shown as follows: Cut out a disk of paper a trifle smaller than a half-dollar. Lay the disk upon the coin and holding the coin horizontally as high as possible, let it drop. The coin clears the way, so to say, so that the paper disk is shielded from the action of the air and falls as fast as the coin, both reaching the table together. When let fall separately, however, the paper flutters down. C. E. LINBARGER.

REFERENCE WORK IN CHEMISTRY FOR SECONDARY SCHOOLS.*

By F. C. Irwin.
Central High School, Detroit, Mich.

The subject of Reference Work in Secondary Schools may be considered under two divisions; first, that required of the pupils; second, the most profitable line of work for the teacher.

In regard to reference work for the students, I think most teachers of elementary chemistry will agree that only a small amount can be done during the first half-year of the course. Now and then a historical note or a more extended account of some application in daily life may be given to advantage, but in general we find the time almost too limited for thorough work in the text and the laboratory work. Then is the time when a great deal of drill is necessary in problems illustrating the quantitative foundation of chemistry, in fundamental laws, laboratory technique, and first general descriptive work and nomenclature of the science. All these essential fundamentals bring in so many new conceptions to the pupil that I doubt the wisdom of scattering his energies to any great extent outside the lines of a good text-book.

After the groundwork is well established, however, considerable systematic reference work will add interest to the subject and give the student a much broader conception of the field of Chemistry. Furthermore, we wish our students to become familiar with the fact that there is a vast literature on the subject. They cannot, of course, become familiar with much of the literature itself, but they can at least know that it exists. Professor Strong, of the Michigan State Normal College, says, "I want my students to know something of the bibliography of Physics." The same is true of Chemistry. It shows the student where he can obtain more accurate and more detailed information than his text-book affords. It gives him an incentive to further study, requires him to dig out facts from the chemical dictionary, and stimulates his imagination and appeals to his ambition by opening up the vast field of industrial chemistry.

I think most of us will grant that reference work is a good thing. The problem resolves itself into two questions, then:

*Read before the Chemistry Section, Central Association of Science and Mathematics Teachers, December 2, 1905.

How can it be accomplished with the greatest economy of the students' time? What is the best line of work to follow?

The first of these questions, the economy of time, was a considerable puzzle with us. We worked it out to to suit our conditions, and I presume each teacher will find it necessary to solve this problem for himself largely. A few words in regard to our method, however, may be of interest. In connection with the study of crystallization, for example, we determine the number of molecules of water of crystallization in copper sulphate or barium chloride. This involves a considerable wait for the heating in the air bath. During this period the student tries the action, qualitatively, of heat on half a dozen other crystalline substances. He is then required to obtain some data for each from the chemical dictionary. We ask him to find the number of molecules of water of crystallization, whether efflorescent or deliquescent, what residue remains after moderate heating, and what the action of water is upon this residue. Some of this data, of course, he obtains from his experiment and verifies from the dictionary; other points he obtains from the dictionary entirely. But the chief value we attach to this particular exercise is that the student must find the names of the materials. For example, he may possibly stumble upon sodium sulphate under the article upon sulphates, but hardly one in ten will be able to find di-sodium hydrogen ortho phosphate. A little drill and instruction will enable him to place his finger instantly upon the proper article for full information. Again, under an experiment in determining the relative solubility of salt, potassium chlorate and potassium nitrate in hot and cold water (stated temperatures), we plot a curve of results, interpret certain points on the curve, and compare results with the Comey's Dictionary of Solubilities. Here I would like to say that secondary schools are in great need of a dictionary less bulky than Watt's, and still complete for information on a wide range of chemical salts.

These two instances, however, illustrate a possible method for doing considerable systematic reference work with small expenditure of time.

As indicated above, some historical references can be used with profit for the beginning of the work. For this Venable's "Short History of Chemistry, published by Heath & Co., at $1.00, is a most usable little book, while for teacher and student alike T. E. Thorpe's "Essays in Historical Chemistry" is a wonderful in-

spiration. Von Meyer's "History of Chemistry" is much more complete than either of these. It traces the development of nearly every department of Chemistry. Thorpe, on the other hand, is biographical. Much of it reads like a romance.

On the whole, however, reference books and reference work will bring best results if arranged to stimulate the interest of the individual student. Keep the books in a convenient place in the laboratory and refer the student to a paragraph while his interest is at white heat, over some unexpected turn of the experiment at hand. From simple experiments in electrolysis, particularly electrolysis of fused sodium hydroxide for metallic sodium, many of the boys will pore over Blount's "Electro-Chemistry." Another class of boys will become especially interested in the manufacture of gas, another in acids, another in explosives, paints, dyes, cements; it is almost impossible to foresee always what line of discussion may be developed, and Thorpe's "Industrial Chemistry" for this work is perhaps more in demand with us than any other book.

Hardly a day passes without some questioning about chemistry of comomn life. For this we have found Lassar-Cohn's "Chemistry of Daily Life," Faraday's "Chemical History of a Candle," and Martin's "Story of a Piece of Coal" can be used to great advantage with students.

For General Inorganic we use Roscoe and Schorlemmer perhaps as much as any one, while Remsen's or Bernthsen's Organic will perhaps be more intelligible to the beginner than Richter-Smith. As a general thing I do not think that formal college texts are of much value for high school students, still we do keep a few of these in the laboratory and our experience is that some of them are in the hands of some of the pupils nearly every day. Among those most used I would mention Remsen's & Newth's for General Chemistry, Prescott & Johnson's and Perkins' for Qualitative Analysis. In fact, we have completely worn out two copies of Prescott & Johnson, one of Freer, one of Remsen, and several sets of chemical tables.

On the theoretical side for students, I do not think we can go much beyond the limits of a good text, but very frequently I have found students who get a great deal of help from "Chemical Theory for Beginners," by Dobbin & Walker, and "Physical Chemistry for Beginners," by Van Deventer.

Along with our own text, Hessler & Smith, we find Newell's

Descriptive, Linebarger's Elementary, Bradbury's and Avery's all very useful for collateral study.

Before closing my list from the pupils' standpoint, I wish to mention the very satisfactory help derived from some periodicals; among them *School Science, Technical World* and the *Scientific American* are before our students every day.

In the second place, reference work or reference books, from the teacher's standpoint, presents such an immense range of literature that we feel at first bewildered. I will mention merely a few books that I know somewhat intimately, and leave the rest for your suggestion and enlargement.

Almost first in helpfulness, I believe, is Smith & Hall's "Teaching of Chemistry and Physics." This contains also one of the best bibliographies of chemistry I have seen. If each book there mentioned could be within the reach of the chemistry teacher he would be well equipped. The teacher should have convenient access to as many reference books as possible. It is a good thing to have them in the city or town library; it is better to have them in the school library, and it is vastly better still to have the most important in the laboratory at hand all the while.

Among the dictionaries we have already mentioned Watts' and Comey. If Thorpe's "Dictionary of Chemical Industry" can be added to these, the reference library will be exceptionally well equipped in this line.

For General Chemistry, every teacher should add a copy of Freer and one of Holleman's "Inorganic Chemistry"; also Newth's and Remsen's college chemistries. In analytical work, Ostwald's "Foundations" stands first for the theoretical side, which is too often neglected in analytical work; next Clowes & Coleman's "Quantitative Analysis" is the best all-around book for analytical methods. The high school teacher will seldom meet an analytical problem which this book does not cover. Still, it is well to be fortified along some special lines also, for but few days pass in which some boy does not produce from his pocket a mineral from a far or near locality, a bottle of well water, a jar of milk, a deposit obtained in his home-made electrolytic experiments, or even a sample of illuminating gas, with the modest request, "Will you help me analyze this?" Now, I do not advocate aimless work followed up merely to satisfy an idle curiosity, but very many of these queries from widely different fields represent real desires for fuller information. Many

times also they will be impossible or impracticable of answering in full, still the line of solution may frequently be pointed out and the student encouraged in taking up systematic analytical work, perhaps in place of testing for gold in iron pyrites. The following books should be at hand: Perkins for Organic Analysis, Leffman and Beam for "Technical Examinations of Water," Hemrel "Gas Analysis," Blair's "Chemical Analysis of Iron." The Year Book of the Department of Agriculture, 1902, gives the analysis of water and interpretation of results, Division of Publication; and the Geological Survey gives methods of mineral analysis.

In the department of Physical Chemistry every progressive teacher will, I think, do some work and considerable reading during the school year. On account of the more recent development of this side of chemistry, it presents all the attractive freshness of a comparatively new field of study, while the experimental part affords great opportunity for the initiative of the worker.

The following books are especially good in this field of the literature:

Jones' "Elements of Physical Chemistry."

LeBlanc, "Electro-chemistry" and Ostwald "'Luther' Physikochemische Messungen" have proved most useful. We should add Walker's "Introduction to Physical Chemistry."

Nernst-Palmer—"Theoretical Chemistry."

Lupke-Muir—"Electro-chemistry."

Jones—"Freezing Point, Boiling Point and Conductivity Methods."

Erdmann—"Inorganic Chemistry."

Ostwald—"School of Chemistry."

Ostwald—"Principles of Inorganic Chemistry" and "Physikalische-chemische Tabellen von Landolt and Bornstein," Berlin, by Julius Springer.

Finally there is an immense range of literature from which to gather miscellaneous information about the things of daily life. Much of this treats of the subject in a popular way, but is none the less valuable for freshening up the instruction and keeping the teacher keen and alert in his work. One of the best means of keeping in the spirit of progressive work is by the reading of a periodical, such as the *Chemical Journal, Journal of Chemical Industry,* or *Science.* The lines of study here indicated will furnish continual inspiration to the conscientious teacher. If the field seems too great, we may recall Professor Smith's remark that, "No preparation is too great for the difficult task of teaching beginning Chemistry."

THE TREATMENT OF LIMITS IN ELEMENTARY GEOMETRY.

By N. J. Lennes,
Wendell Phillips High School, Chicago.

SECTION III.

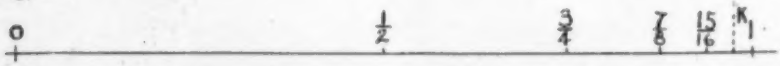
Constructive Treatment of Limits in Elementary Geometry.

PART I. THEORY OF LIMITS.

It is a *sine qua non* that shoddy proofs must go. Clearness and honesty of thought is the ideal of mathematics if of any field of intellectual activity. If it should not be found possible to formulate a solid treatment of limits adequate to meet the needs of elementary geometry and at the same time sufficiently simple to be understood by the pupil, then either the topics where limits is now used must be omitted or a comprehensible treatment must be devised without the use of limits. In case the latter alternative were chosen one might consider only those segments, angles etc., which are commensurable. Of course, it would have to be stated explicitly that such treatment is not complete, but that we assume the theorems without proofs in case the segments, etc., are incommensurable. The length of a circle might be considered as the perimeter of a regular polygon of, say, one million sides, etc. Our results would then be accurate far within the limits of observation. At every step we should have to be perfectly clear as to what we are doing. The treatment could easily be arranged so as to be entirely logical throughout.

From considerable experience in secondary instruction the writer is led to believe, however, that it is possible to construct a theory of limits applicable to elementary geometry which shall be entirely rigorous and at the same time will be within the reach of the average pupil. It remains, therefore, to attempt a constructive treatment of limits in connection with ratio and proportion, length of a circle, etc. We develop here only so much of the general theory of limits as is required for these purposes.

On a straight line mark a certain point O and lay off to the right of this point a sequence of points such that the distances of the successive points from the point O are 1/2, 3/4, 7/8, 15/16.



If this sequence of points is continued ever so far it will never reach the point 1. On the other hand if we select any point whatever "K" to the left of 1, no matter how near to 1, we shall always be able to go far enough in the sequence of points denoted by the sequence of numbers 1/2, 3/4, 7/8, to obtain a point between "K" and 1. For the sake of brevity we will speak of all the points which can be obtained by continuing the sequence 1/2, 3/4, 7/8, indefinitely, as the "set of points (P)" or simply (P). The point 1 has two definite relations to the set (P).

(a). There is no point of the sequence (P) to the right of 1.*

(b). For every fixed point "K" to the left of 1 there are points of (P) to the right of "K".

The following is the kernel of the whole thing and should be clearly understood: The point 1 is the *only point* of the whole line such that both (a) and (b) are true of it. (a) is true of every point to the right of 1 but (b) is not. (b) is true of every point to the left of 1, but (a) is not. It follows, therefore, that while the points of the sequence (P) merely *approach* 1 they nevertheless serve to determine the point 1 as *definitely* as if the number 1 were used.

Definition: A sequence of objects which cannot be completely counted no matter how far the counting proceeds is said to be an infinite sequence.

Not every infinite sequence of points serves to single out a definite point in the same manner that 1/2, 3/4, 7/8, singles out 1. Thus 1, 2, 3, fails to do so as does also the sequence 1, 2, 1, 2, In what respects do these last two sequences differ from the sequence 1/2, 3/4, 7/8? In the sequence 1, 2, 3, the numbers grow large beyond all measure. Such sequence is said to be *unbounded,* while the sequence 1/2, 3/4, is *bounded.* (That a sequence is bounded means that there is some number greater than every number of the sequence). The sequence 1, 2, 1, 2, 1 is said to be *oscillating* since the terms grow larger and then smaller, etc., while the sequence 1/2, 3/4 is *non-oscillating.* A non-oscillating sequence cannot increase and then decrease, or *vice versa,* but it need not increase with every term. Thus 1/2, 3/4, 3/4, 5/6 is a non-oscillating sequence as is also 1, 2, 3, 3, 4.

* We might say with equal accuracy that 1 is to the right of every point of (P). This is not done because of a desire to be as near as possible to the more general discussion of advanced mathematics.

We wish now to concentrate our attention on infinite sequences which are *both bounded and non-oscillating*. In the sequence 1/2, 3/4, already considered, 1 is said to be the *least upper bound* of the sequence. This statement includes precisely the two statements (a) and (b) on page ??. (a) 1 is an upper bound i. e. there is no point of (P) to the right of 1. (b) 1 is the *least* upper bound which means that no point to the left of 1 is an upper bound. Similarly the point 2 is the least upper bound of the sequence 1, 1½, 1¾. The sequences which we have considered are extremely simple and their least upper bounds are points with which we are already acquainted.

If we attempt to extract the square root of 2 we obtain a sequence 1, 1.4, 1.41, 1.414, 1.4142. This is evidently a non-oscillating bounded sequence. Does it likewise have a least upper bound? Since we know there is a definite point on the line which represents the square root of 2, the answer is yes. We now adopt the following axiom:

Axiom of Continuity. *Every bounded, non-oscillating sequence of points has a least upper bound.** Since a number is to correspond to every point on the line, we use the equivalent axiom about numbers.** .*Every bounded non-oscillating sequence of numbers has a least upper bound.****.

This axiom of continuity goes right to the heart of the matter of limits as needed in elementary geometry. This axiom is likewise of the inmost importance in the most critical parts of modern higher mathematics.

We shall often refer to a point as being to the right or to the left of another point. If these points have numbers corresponding to them we always fix matters so that we may say of two points that one which has the greater number corresponding to it is to the right of the other. In this sense we may speak of an increasing sequence of points.

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* It is assumed that at this point the notions of order on a line—betweenness and direction—have been given definite logical meaning.

** Space does not permit a fuller statement about what is called the one-to-one correspondence between points and numbers. The idea is simple, however, and in its elementary form can easily be made accessible to the student of geometry.

*** This axiom of continuity is by no means trivial as may appear at first sight, and it is easy to prove that it cannot be proved by means of the remaining axioms of Euclidean geometry.

Lemma. If (P) and (P′) are increasing sequences of points (numbers), and if B and B′ are the least upper bounds of these sequences, then if B is to the right of (greater than) B′, it follows that there is at least one point (number) of the sequence (P) to the right of (greater than) all points (numbers) of the sequence (P′).

The proof follows immediately from the definition of least upper bound.

PART II. APPLICATIONS.

We now define after the manner of our elementary texts: Two segments (numbers)are commensurable if there exists a unit-segment (number) which will exactly measure both of them. The ratio of two commensurable segments (numbers) is the ratio of the numbers of times the unit of common measure is contained in each.

Now consider two incommensurable segments AB and A′B′. On segment AB lay off AC_1 such that A′B′ and AC_1 are commensurable, having a common measure m_1. AC_1 is also to be such that C_1B is less than the unit-segment m_1. Let the ratio of A′B′ and AC_1 be R_1. Proceed in this manner using a sequence of unit-segments m_1, m_2, m_3, which becomes as small as we please as we pass on in the sequence. Then we obtain a sequence of ratios R_1, R_2, R_3. This sequence can easily be proved to be non-oscillating and bounded and hence, by the axiom of continuity has a least upper bound R. Suppose instead of using the sequence of units m_1, m_2, m_3, we had used another sequence n_1, n_2, n_3. We should then obtain a different sequence of ratios R_1', R_2', R_3'. By means of the lemma it follows that the least upper bound R′ of this sequence is identical with R.* The number R is *defined* as the ratio of the segments AB and A′B′. *The real point is that we obtain the same number R no matter what sequence of units of measure we use,* it being understood that the units shall decrease indefinitely as we pass on, and that BC_n shall be less than m_n for every *n*.

The ratios of incommensurable angles and of incommensurable arcs of the same or equal circles are defined in precisely the same manner.

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* It may not be possible to write down the number R as an integer, or a fraction, whose terms are integers. R is a real definite number for all that. We are already familiar with such numbers, $\sqrt{2}$, $\sqrt{3}$, $\sqrt[3]{4}$, etc.

We now prove the following *theorems* (*"A straight line parallel to one side of a triangle divides the remaining sides proportionally."*). *Proofs*. Let the line in question be parallel to the sides AB of the triangle ABC and cut the sides AC and BC in the points D and E, respectively.

The number representing the ratio between the segments AD and AC is obtained according to the method described on page ?? by means of a certain sequence of points D_1, D_2, on the segment AD such that AD_1, AD_2 and DC are commensurable. The corresponding sequence of ratios R_1, R_2 defines the ratio R of the segments AD and DC. The lines through the points D_1, D_2, parallel to AB meet the segment BE in the points E_1, and E_2, then BE_1, BE_2 and EC are commensurable and we obtain an increasing sequence of Ratios R_1', R_2', which defines the ratio of R′ of BE and EC. But by the theorem in the commensurable case $R_1'=R_1$, $R_2=R'_2$, etc. Hence the ratios R and R′ are defined by the *same sequence* and are identical, which proves the theorem.*

The scheme of proof is essentially the same whenever we wish to prove two incommensurable ratios equal. If we can show that the ratios in question are determined by the same sequence we shall have it.

Let us now consider the circumference of the circle. When we speak of the *length* of a straight line-segment we mean the ratio of this segment to a unit-segment. We have just seen how to define this ratio in all cases. The case of the *length* of a circle is not so simple. We cannot even *begin* to measure the circle by means of a straight line measuring-stick. Our measure will not coincide with the thing to be measured any part of the distance.

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* To a person not accustomed to this kind of argumentation this proof may seem essentially that given in the current texts including the use of the theorem that two equal variables have equal limits. But this is not the case. The argument here is: (1) A certain kind of sequence (which is the kind we have to deal with in geometry) *determines one number.* This is the immediate consequence of the axiom of continuity. (2). Two different sequences having certain relations (the kind of relations that exists among the sequences with which we have to deal in geometry) determine the same number. (This was proved as a lemma). At this stage we are first unable to *speak of the ratio* of two incommensurable segments. (3) We prove that the two ratios which we wish to prove equal are determined by the same sequence.

Hence we proceed as follows: Inscribe two regular polygons in a circle such that one has twice as many sides as the other. The polygon with the greater number of sides has the greatest perimeter. Consider a sequence of regular inscribed polygons such that any one polygon of the sequence has twice as many sides as the one which precedes it. Then we obtain an *increasing* sequence of perimeters p_1, p_2. Further, this sequence is *bounded* since the perimeter of any convex inscribed polygon is less than the perimeter of a circumscribed polygon. Hence the sequence, p_1, p_2, determines a definite number, p. We now make use of the following theorem: "If P′ and P″ are two convex polygons inscribed in the same circle and if the longest side of P′ is shorter than the shortest side P″ then the perimeter of P′ is greater than that of P″."* By means of this theorem and the lemma on page ?? we have at once that any sequence of perimeters obtained by inscribing a sequence of polygons, such that the largest side of each polygon decreases indefinitely as we pass on in the sequence of polygons, will define exactly the number, p, defined also by the sequence p_1, p_2. This number p we *define* as the *length* of the circle.

The futility of attempting to *prove* that the number p is the length of the circle should now be apparent. The expression "length of circle" has no logical existence before it is defined and if we attempt to define it (otherwise than by a set of axioms) we are inevitably driven to make the above definition in substance if not in form.

To proceed from this point we adopt the following definition: If p_1, p_2, is a sequence of numbers defining a number p, then the number defined by the sequence cp_1, cp_2, cp_3, ... is the product cp.

Then we can prove such theorems as the following: "*Two circles have the same ratio as their radii.*"

The theory of plane areas of curvilinear figures, of volumes, and of surfaces of curvilinear solids is developing in a manner analogous to the above.

The sequence of numbers which we consider under our axiom may be decreasing instead of increasing. In that case we specify the existence of a unique greatest lower bound instead of a least upper bound of the sequence.

* This theorem can be readily proved. In fact practically equivalent theorems are proved in most of our text-books.

To illustrate, consider the area of a sphere. Suppose we start with a cube whose edges are 10 centimeters. By continual removal of vertices we approach a sphere whose diameter is 10 centimeters. Remove the 8 vertices of the cube by means of planes tangent to the sphere. We then obtain a 14-sided figure, the area of whose surface is less than 600 centimeters. Denote this area by s_1. Again remove all the vertices of this figure by means of planes tangent to the sphere thus obtaining a new figure with surface area s_2. Proceeding in this manner we obtain a decreasing sequenee s_1, s_2, s_3 which is bounded and hence defines a definite number. This number we *define* as the area of the sphere.

A *proof* that this is the area of the surface is as impossible here as in the case of the circle and for exactly the same reasons. It is then a matter of *proof* that the area *so defined* is equal to $4\pi r^2$, etc.

CULTURE COURSE. (III.)

By T. M. Blakslee.
Central City, Iowa.

(Continued from the December Number, p. 755.)

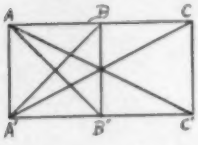
Part III, Sec. I. Introduction.

(a). *Simple Equation.* Nothing new needed.

(b). Linear Equations with *two* unknowns.

$ax+by=0$ (1) $a'ax+a'by=a'c$
$a'x+b'y=c'$ (2) $aa'x+ab'y=ac'$ $y=\dfrac{ac'-a'c}{ab'-a'b}$

The numerator is the algebraic sum of the products of the diagonal elements in the entire rectangle The denominator is the like sum for the left partial rectangle. The sign of the product is opposite to that of the slant of the diagonal.



Solve mentally:

(1). $2x+2y=17$, $4x+y=16$

(2). $2x+y=9$, $5x+3y=25$

(3). $\dfrac{10}{x}+\dfrac{9}{y}=8$, $\dfrac{8}{x}+\dfrac{15}{y}=9$

(4). $\dfrac{a}{x}+\dfrac{b}{y}=c$, $\dfrac{b}{x}+\dfrac{a}{y}=c$

(5) $2mp+nq=k$, $3mp-5nq=1$;

Solve, first for p and q, then for m and n.

(c). Linear Equations is any number of unknowns.

$A_1x_1+B_1x_2+C_1x_3+\ldots\ldots \quad L_1x_n=M_1 \quad (1)$

$A_2x_1+B_2x_2+C_2x_3+\ldots\ldots \quad L_2x_n=M_2 \quad (2)$

$A_nx_1+B_nx_2+C_nx_3+\ldots\ldots \quad L_2x_n=M_n \quad (n)$

By (1) $x_1=\frac{M_1}{A_1}-\frac{B_1}{A_1}x_2-\frac{C_1}{A_1}x_3\ldots\ldots \quad \frac{L_1}{A_1}x_n$. Hence

$(A_1B_2-A_2B_1)x_2+(A_1C_2-A_2C_1)x_3+\ldots\ldots=A_1M_2-A_2M_2 \quad (2)$

Similarly for $(3)^1 \quad \ldots\ldots(n^1)$.

Definition: $\begin{vmatrix} a_1 , & b_1 \\ a_2 , & b_2 \end{vmatrix}=a_1b_2-a_2b_1.$

Solve :

(1) $\begin{cases} x+2y+z=12; \\ 2x-3y+4z=11; \\ x+5y+2z=3. \end{cases}$ (2) $\begin{cases} -7y+2z=-13; \\ y-3z=-9; \\ z=4. \end{cases}$

(3) $\begin{cases} 2x+3y+z=11; \\ 5x+y+z=13; \\ x+2y+3z=14. \end{cases}$ (4) $\begin{cases} a_1x+b_1y+c_1z=d_1; \\ a_2x+b_2y+c_2z=d_2; \\ a_3x+b_3x+c_3z=d_3. \end{cases}$

APPLICATIONS.

Problem—*To find the condition that three linear equations containing but two unknowns shall be consistent.*

Given $a_1x+b_1y+c_1=0 \quad (1)$

$a_2x+b_2y+c_2=0 \quad (2)$

$a_3x+b_3y=c_3=0 \quad (3)$

$a_1x+b_1y=-c_1 \quad (1)'$

$a_2x+b_2y=-c_2 \quad (2')$

$\begin{vmatrix} a_1 , & b_1 , & c_1 \\ a_2 , & b_2 , & c_2 \\ a_3 , & b_3 , & c_3 \end{vmatrix}=0.$

Test the following:

$2x+3y+1=0 \qquad 2x+3y+4=0$

$3x+4y+1=0 \qquad 3x+2y+4=0$

$4x+5y+1=0 \qquad 4x+3y+2=0$

(d) *Second Degree,* One Unknown (Quadratics).

Type Equation: $x^2+2px=q \quad (1)$

$x^2+2px+p^2=q+p^2 \quad (2)$

$x_1=-p+\sqrt{q+p^2}, \; x_2=-p-\sqrt{q+p^2} \quad (3)$

$x_1+x_2=2p, \; x_1x_2=-q \quad (4)$

(x—a) (x—b) = 0 has a and b as roots, i.e., these numbers, (when substituted for x), satisfy the equation.

Form 10 equations with special values for a and b and test by solving.

Second Type: $ax^2+bx+c=0$ gives

$$x=\frac{-b+\sqrt{b^2-4ac}}{2a}$$

The roots are (1) Real and Unequal, (2) Real and Equal, (3) Imaginary, (i.e., transverse) according as

$$b^2 \gtrless 4ac.$$

Polynomials in the Determinant Form.

$\begin{vmatrix} a_1 , & b_1 \\ a_2 , & b_2 \end{vmatrix} = a_1b_2-a_2b_1$ is of the second order.

$$\begin{vmatrix} a_1 , & b_1 , & c_1 \\ a_2 , & b_2 , & c_2 \\ a_3 , & b_3 , & c_3 \end{vmatrix} = \begin{matrix} a_1b_2c_3+a_2b_3c_1+a_3b_1c_2 \\ -a_1b_3c_2-a_2b_1c_3-a_3b_2c_1 \end{matrix}$$ of third order.

The science of these abbreviated forms is the Theory of Determinants.

All we need to observe here is that the first member denotes the algebraic sum of the products of three elements, one, and but one, being taken from each row and each column. The sign of a product is *contrary* to the slant of the ray of two of its elements.

$$\begin{vmatrix} a_1, & b_1, & c_1 \\ a_2, & b_2, & c_2 \\ a_3, & b_3, & c_3 \end{vmatrix} \quad \text{Ill.} \quad \begin{vmatrix} 3, & 5, & 7 \\ 4, & 2, & 1 \\ 1, & 6, & 7 \end{vmatrix} = 43$$

+	−
42	14
168	19
5	140
215	− 172 ≅ 43

(To be continued.)

A PECULIAR ACTION OF THE RADIOMETER.

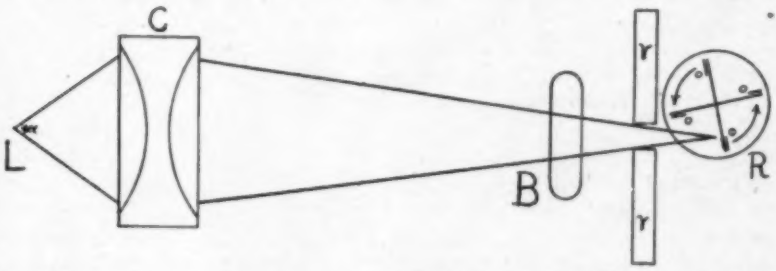
By C. F. Adams,

Central High School, Detroit, Mich.

While arranging apparatus at one time to show selective absorption I happened to place a radiometer at the focus of light coming from the lantern so that the light was incident on the bright side of the vanes only. I was surprised to see it revolve in a direction opposite to the usual direction. Recently I repeated the experiment arranging the apparatus as shown in the figure in which C represents the condenser of a lantern, L an arc light, and R the radiometer. Two boards about 2 cm thick, were placed at r r with a space between them of about 1 cm. I had

two flat bottles, one filled with water and the other with a very dark solution of iodine in carbon bisulphide. I shall call the bottle containing water, bottle No. 1, and the other No. 2. B represents the position of a bottle when in use.

With no bottle in place the direction of rotation was as shown by the arrow, opposite to the usual direction. With bottle No. 1



in place, when the greater part of the heat is cut off, the rotation is in the same direction as before and apparently at about the same rate. When bottle No. 2 is substituted for No. 1, the radiometer stops its rotation in the direction first given and turns in the opposite direction. In this case the heat at the radiometer is nearly as intense as when no bottle is in place. With both bottles in place, the direction of rotation is still the same as when no bottle is used, but very slow.

It is easy to see why the rotation is as described with no bottle in place. The radiation falls only on the bright surfaces of the vanes and hence those surfaces become warmer than the blackened surfaces; but why the rotation is reversed upon the interposition of bottle No. 2, is not so easily explained. I have thought that perhaps it was due to fact that the long waves transmitted by the iodine solution passed through the mica of the vanes to the lampblack which absorbed them, so that the dark surfaces became warmer than the bright sides; but there are serious objections to this explanation. I have no other to offer, however. It may be that the phenomenon is one well known and has been explained, if so I am not aware of it.

One of the most comprehensive educational reports published in North America is that of October, 1905, by the Superintendent of Education of the Province of Nova Scotia. It is full of information which is useful to all teachers. It shows that the schools of this particular part of Canada are ably and well managed. The report recommends for the use of teachers SCHOOL SCIENCE AND MATHEMATICS.

DEPARTMENT OF METROLOGY. NOTES.

English Measures Unknown in Europe.—The Boston Advertiser says: "Few people realize what it means to take an American-made automobile to Europe, for it is utterly impossible to replace a part while there. The reason for this is that throughout Europe the metric system of measurement is used and all threads are measured by metric system." Of course this is true of other American machinery and paraphernalia of traveling. Amateur photographers have been among the most sufferers, as metric plates do not fit non-metric cameras. R. P. W.

A New Calendar for France.—The world owes France a debt of gratitude for the Metric System. The failure of that nation to maintain the decimalization of time units is well known. Oct. 5, 1792, a decimal calendar was decreed by the Assembly, according to which 100 seconds made a minute, 100 minutes an hour, 10 hours a day, 10 days a week. This went into compulsory effect Nov. 24, 1793. Its abolition was accomplished by stages, the division of the day being legally abrogated April 7, 1795, with barely a year and a half of existence. The 10-day week was not repealed till 7 years later, April 8, 1802, and in fact the entire new calendar of months was not actually abolished till Jan. 1, 1806. The whole decimal plan came near practical annihilation when, Feb 12, 1812, Napoleon decreed the abondment of the entire decimal system of weights, measures and everything else. Not until July 4, 1837, was the great dictator's myopic edict repealed, and the Metric System reestablished on a compulsory basis beginning Jan 1, 1840

All this is history, and the decimalization that has come down to the present time is mainly confined to weights, measures and money. Now comes a Frenchmen, M. Camille Flammarion, well known astronomer, who proposes to recommend to the Chamber of Deputies a bill to make compulsory a new calendar of his own device. The change proposed is not a drastic one, not even decimal, and it has relation only to months, and days. But it is vastly superior to the present motley arrangement in that the year would begin at a natural point of time, and months be divided according to a definite system. Flammarion's year is to begin at the Vernal Equinox, March 21, and be divided into quarters, each quarter into three months, two of which contain 30 days, the third 31 days, 364 days in all. The 365th day belongs to no month and is to be a fete day. Leap years have two such days. One great advantage to this calendar is that every year begins on the same day of the week, as also does each corresponding month of successive years.

It cannot be expected at this day that so radical a change will be wrought even by France, desirable as the change might be, for it would be opposed from a religious even more than a secular standpoint. This scheme of beginning and dividing the year, while credited to Flammarion, has probably been thought out by thousands of people since the Emperor Augustus disarranged the days of the month in his zeal to have the month named from him contain as many days as that of his illustrious predecessor Julius, to whom we owe our calendar. R. P. W.

DEPARTMENT OF ZOOLOGY.

COLLEGE ENTRANCE OPTION IN ZOOLOGY.

At the Washington meeting of the scientific societies three years ago the newly organized American Society of Zoologists voted to appoint a committee to consider the question of college entrance options in zoology, particularly with reference to the work of the College Entrance Examination Board. The committee appointed consisted of Professor Cornelia M. Clapp, of Mt. Holyoke; Professor E. G. Conklin, of the University of Pennsylvania; Professor C. W. Hargitt, of Syracuse University; Professor J. S. Kingsley, of Tuft's College; and Professor M. A. Bigelow, (Chairman), of Teachers College, Columbia University. This committee submitted a report to the Eastern Branch of the Society of Zoologists in December, 1904; and, without opposition, it was voted to "approve" the report. Since then it has been adopted (Nov. 11, 1905) as a requirement by the College Entrance Examination Board so far as the one-point (full year) in zoology is concerned; and now for the first time zoology is recognized among the secondary-school subjects in which the Board will give examinations.

The preliminary report of the committee, published in SCIENCE for December 16, 1904, is with the exception of a few minor changes the same as the following, which contains the one-point option as adopted by the College Entrance Examination Board.

INTRODUCTION.—Believing that zoology should have a place in general liberal education and recognizing that for the great majority of citizens formal education must end with the secondary school, it is the opinion of this committee (1) that this science should be taught in high schools for the benefit of pupils who will have no other opportunity of acquiring general knowledge of animals; and (2) that zoology thus taught from the point of view of general secondary education should have recognition as a college-entrance option, in order that pupils who can not decide to go to college before the close of high-school work may not be held deficient in credits because zoology was elected. But, although thus recognizing zoology from the standpoint of secondary education, this committee wishes to emphasize the opinion that zoology is not one of the most desireable subjects as preparation for college; and that the physical sciences should first of all be recommended to pupils who expect to go to college.

In reaching these conclusions the committee has not failed to consider the value of general acquaintance with common animals and with the essential principles of the elementary physiology and hygiene of the human body, but these are commonly taught in the years below the second of high school, and work in these lower years does not in the case of other subjects closely concern the question of college-entrance options.

These preliminary statements will make it clear that the following suggestions for a scheme of college-entrance credits in zoology are intended by this committee simply to provide for crediting the zoology which should be elective in every good general high school, but not to advocate the subject as one which from the college standpoint is desirable

in preparation for college. In other words, this committee is simply recommending that zoology studied as part of a liberal secondary education intended primarily to prepare for life should be recognized as preparing for entrance to college (the minor question of preparation for entrance to college courses in zoology being here laid aside.)

General Statement of Options—(*a*) One-point option. To count as one unit or point in thirteen to fifteen required for entrance to college. This should consist of one year of at least five hours per week devoted to study of zoology, as indicated in outline of course given below.

(b) Half-point option (one point in biology.) To count as one half unit in thirteen to fifteen required, only when a half unit of botany taken in the same continuous course is offered to complete a full unit in biological science. A half year of zoology independent of botany should not be accepted.

A two-point option is not recommended, because more than one full year in zoology is extreme specialization which no secondary school can properly undertake.

Outline for the One-point Option in Zoology.—The following outline includes the principles of zoology which are indispensable to a general survey of the science. It is not intended to indicate order of study of the topics—this must be left to the teacher and the text-book. With little modification the courses presented in high-school books on zoology cover the ground outlined below.

1. The general natural history—including general external structure in relation to adaptations, life histories, geographical range, relations to other plants and animals, and economic relations—of common vertebrates and invertebrates so far as representatives of these groups are obtainable in the locality where the course is given. The types suggested are a mammal, bird, lizard, snake, turtle, newt, frog, dogfish or shark, bony fish, clam, snail, starfish, earthworm, planarian, hydra, sea-anemone, paramoecium. In the case of arthropods, pupils should become familiar with common crustaceans, spiders, myriapods, and insects representing at least five orders. Actual examination of common animals with reference to the above points should be supplemented by reading giving natural-history information. It is not expected that there will be time for making extensive note-books on the natural-history work; rather will the work in this line take the form of laboratory demonstrations. So far as time permits drawings and notes should be made. The note-book mentioned below should contain drawings on the external structure of at least four animals in addition to those mentioned under Section 3, preferably two insects, a mollusk, and a second vertebrate.

2. The classification of animals into phyla and leading classes (except the modern subdivisions of the worms) and the great characteristics of these groups. In the case of insects and vertebrates the characteristics of the prominent orders. The teaching of classification should be by practical work so as to train the pupil to recognize animals and to point out the chief taxonomic characteristics. The meaning of species, genera and larger groups should be developed by constructive practical work with the representatives of insect or vertebrate orders.

3. The general plan of external and internal structure, not the anatomical minutiæ, of one vertebrate (preferably frog or fish) in general comparison with human body; an arthropod (preferably a decapod); an annelid (earthworm or Nereis); a cœlenterate (hydroid, hydra or sea-anemone); a protozoon (a ciliate, and amœba when possible). In place of any of above types not locally available there may be substituted a second vertebrate, an insect, a mollusc or an echinoderm. Tissues (optional, recommended as *desirable*, but not to be required in examinations) should be examined first with the unaided eye, in such a structure as a frog's leg, and then with a microscope demonstrate the relations of cells and intercellular substance in epithelium and cartilage; and, if possible, in other tissues. The functions of the chief tissues and their positions in the body of a vertebrate should be pointed out.

4. (*a*) The general physiology of above types, involving the essentials of digestion, absorption, circulation, respiration, cell-metabolism, secretion, excretion and nervous functions. This should apply comparatively the essentials of elementary work in human physiology. Demonstrations and experiments, such as are suggested in high-school text-books on human physiology, should be introduced, or recalled if not previously well presented in elementary physiology, in connection with the discussion of the chief functions. As far as practicable structure and function should be studied together.

(b) Comparison of the general life-processes in animals and plants (in connection with botany if zoology is first studied).

5. The very general features of a sexual reproduction of a protozoon (preferably *Paramoecium*); alternation of generations in hydroids; reproduction and regeneration of *Hydra*; the very general external features of embryological development in a fish or frog; and (*optional*) the general cellular nature (not centrosomes and the like) of germ-cells, fertilization and cell-division in developing eggs should, as far as possible, be demonstrated and briefly described. Also, the most interesting features of development should be pointed out in the case of other animals studied.

6. The prominent evidences of relationship, suggesting evolution, within such groups as the decapods, the insects and the vertebrates, should be demonstrated. A few facts indicating the struggle for existence, adaptation to environment, variations of individuals and man's selective influence should be pointed out; but the factors of evolution and the discussion of its theories should not be attempted.

7. (*Optional*) Some leading facts regarding the epoch-making discoveries of biological history and the careers of such eminent naturalists as Darwin, Huxley, Pasteur and Agassiz should be presented.

The above outline of a course in general zoology should be developed on the basis of a course of laboratory study guided by definite directions. This should be supplemented by the careful reading of at least one modern elementary text-book in general zoology. At least two-thirds of the time should be devoted to the practical studies of the laboratory If good nature-studies have not preceded the course in high-school zoology, pupils should be encouraged to do supplementary work in the line of natural history. A note-book must be submitted, properly certified by the teacher, at the time

of the examination. It should contain carefully labeled outline drawings of the chief structures studied anatomically (Section 3) and the drawings mentioned under natural-history (Section 1); and also notes on demonstrations and in explanation of drawings. All drawings and notes should be dated when made; and the book should be indexed. The note-book will be graded as one-third of the examination.

The question whether a course as outlined above should admit students to the second college course in zoology is one which must be answered by each college for itself. It is quite unimportant so far as accepting an entrance option in zoology is concerned, for the very few pupils who study the science in high school and later in college have special interests which make adjustment of their college work easy.

Outline for the Half-Point Option.—This is suggested as preliminary to a joint conference with the committee on botany, and looks towards the recognition of a year in biology. It was not considered by the College Entrance Examination Board. (1) The general natural history specified above. (2) The classification of animals specified above. (3) The general internal structure of one vertebrate and a decapod or annelid. (4) The physiology of these two animals along the lines suggested above, with special application to the functions of the human body, and comparison with the general functions of plants. (5) The general external embryology of frog as suggested above.

NOTICE.

When pupils are determining resistance by means of the slide wire bridge, time and labor are saved by using the tables prepared by Ostwald which give the valve of a for all valves of $\frac{a}{1-a}$ from 0.001 to 0.999 meters.

For sale by SCHOOL SCIENCE AND MATHEMATICS at the rate of $1.50 per hundred or ten for 25 cents post paid.

CHANGE IN WHOLSALE PRICE OF TOPOGRAPHIC MAPS.

PUBLISHED BY THE UNITED STATES GEOLOGICAL SURVEY.

On and after July 1, 1905, the wholesale rate on all topographic maps will be advanced one-half; that is, the price of the standard or small size map sheet will be increased from $2.00 to $3.00 per hundred, and the price of the large maps will be increased proportionately. The discount will be 40 per cent. of the retail price when 100 or more standard topographic atlas sheets, or their equivalent in other maps, are ordered. In no case can orders be filled at the wholesale rate unless the sum remitted solely for topographic maps amounts to at least $3.00. No change will be made in the retail rate.

No order received after June 30, 1905, will be filled at the old rate, no matter when dated or when mailed.

A wholesale rate immediately effective has been established for topographic and geologic folios, whenever the sum transmitted solely for folios at the wholesale price amounts to at least $5.10; that is, a discount of 40 per cent. will be given whenever 34 or more folios that sell singly

for 25 cents, or their equivalent in higher priced folios, are ordered at one time.

Attention is called to the fact that folios and topographic maps can not be combined in a single order at wholesale rates unless the sum remitted for each is at least the minimum stated above.

It is a pity that this notice could not have been sent out to educational institutions, or at least to editors of educational journals that they might notify those interested before the date of advance in price, instead of some months after that date. The retail price of the U. S. maps has been about the same as the wholesale price in small quantities for the special educational editions of the English Ordinance Survey Maps. The wholesale price of the U. S. maps is now, however, about three times the price (to educational institutions) for large numbers of the English Ordinance Survey maps. This change by the United States survey seems a step backward. The only compensation is the reduced price in the wholesale price of the folios.

Asbestos and Its Uses.—Asbestos, being one of the best fire-resisting materials known, is becoming of considerable importance in engineering work. This mineral is found in all quarters of the globe. The only kinds so far found possible for what may be termed universal commercial purposes are the Italian and Canadian varieties, although the Russian and the South African specimens have their uses in a more limited field of operation. The Italian and Canadian varieties contain about one-third magnesia and about forty per cent. of silica, the other constituents being iron, potash, soda and alumina. The peculiar characteristics of this material are that it is fibrous, fireproof, an insulator, and a non-conductor of heat, and is also acid-resisting to a great degree. In working up the material, the treatment varies according to the quality of the mineral and the purpose for which it is intended. The blocks of fibre are crushed and opened in machines, in such a way as not to destroy the fibre. They are then passed to shaking machines, where the long fibre suitable for spinning is separated from the short. The long fibre is then passed through the carding and condensing department, and from here on the treatment is similar to that followed in a textile factory, though the peculiar character of the material makes special machinery and treatment necessary. The shorter fibre, suitable for making millboard and paper, is beaten up in pulp machines and treated in much the same way as paper pulp. The asbestos found in South Africa has a blue fibre, and differs from the Italian and Canadian varieties, in that the magnesia is replaced largely by oxide of iron, of which it contains forty per cent. Silica forms fifty per cent of the material, the other constituents being alumina and magnesia. The fibre in this material is longer than in the Canadian. It is also stronger and lighter, and is not as easy to manufacture. Also, on account of the quantity of the oxide of iron, it is not suitable for many purposes for which the other is used. Asbestos is also found in the form of powder, this kind occurring in Italy. The powder is worked up into various kinds of fireproof paint.—*Abstracted from the Electrical Review (London), April* 29.

REPORT OF COMMITTEE ON "AMERICAN SOCIETY OF TEACHERS OF MATHEMATICS AND THE NATURAL SCIENCES."

[The members of the committee have assented to a revision of name as adopted at their first meeting. F. T. Jones prefers The American Society of Teachers of the Natural Sciences and Mathematics.]

The committee met at the Coleman House, Asbury Park, N. J., July 15, 1905. There were present: Thomas S. Fiske, Columbia University, [Chairman]; Clarence E. Comstock, Bradley Polytechnic Institute; E. R. Hedrick, University of Missouri; Franklin T. Jones, University School, Cleveland; W. H. Metzler, Syracuse University; Edgar H. Nichols, The Brown and Nichols School, Cambridge. Franklin T. Jones was elected Secretary.

The following resolutions were adopted:

That the name "National Society of Teachers of Mathematics and Science" shall be recommended. [By agreement among the members of the committee, with the exception noted in the heading of this report, the name "American Society of Teachers of Mathematics and the Natural Sciences" is recommended instead.]

That the first meeting of the society shall be held at the same time and place as the next meeting of the National Educational Association.

That, for this meeting, provision shall be made for a general session and for sectional meetings in Mathematics and the Natural Sciences.

That the chairman and secretary of this committee shall be authorized to draw up a circular addressed to associations of teachers of science, of mathematics and of teachers of science and mathematics throughout one or more states or territories asking them to participate as members of the National (American) Society at its first meeting.

That the associations addressed be asked to define their territory.

That no one shall be eligible to membership in the National (American) Society unless he is a member in good standing of one of the associations to which this invitation is addressed.

That the chairman shall be requested to draw up a constitution to be submitted to the associations addressed which shall be sent to each member of the committee and as soon as it is unanimously approved by the members of the committee, shall be recommended as the constitution of the National (American) Society. [It is assumed that final action on the constitution must be taken by the members present at the first meetings. Proposed changes will be published by the committee from time to time.]

That the chairman and secretary shall be instructed to find the terms on which the society can obtain suitable control of a journal—preferably "SCHOOL SCIENCE AND MATHEMATICS"—as the organ of the National (American) Society.

In accordance with the above resolutions, the chairman, with the approval of the members of the committee, has drawn up a constitution which is hereby submitted to the following associations:

Central Association of Science and Mathematics Teachers.

Association of Teachers of Mathematics in the Middle States and Maryland.

Pacific Coast Association of Chemistry and Physics Teachers.

New England Association of Chemistry Teachers.

Eastern Association of Physics Teachers.

Mathematical and Physical Section of the Ontario Educational Association.

New England Association of Mathematics Teachers.

Natural Science Association of the Ontario Educational Association.

Missouri Society of Teachers of Mathematics.

Association of Ohio Teachers of Mathematics and Sciences.

Science Section of the Michigan Schoolmasters' Club.

Indiana Science Teachers' Association.

Mathematical Section of the California Teachers' Association.

Physical Science Section, Nebraska State Teachers' Association.

Mathematical Association of Washington.

Kansas State Mathematics Teachers' Association.

Texas State Mathematics Teachers' Association.

Louisiana State Mathematics Teachers' Association.

[If any associations which should be entitled to be in this list are omitted, their officers will confer a favor by addressing the secretary of the committee, Franklin T. Jones, University School, Cleveland, Ohio.]

DRAFT OF A CONSTITUTION FOR THE AMERICAN SOCIETY OF TEACHERS OF MATHEMATICS AND THE NATURAL SCIENCES.

ARTICLE I. NAME.

This society shall be called The American Society of Teachers of Mathematics and the Natural Sciences.

ARTICLE II. OBJECTS.

The objects of this society shall be

(1) To improve the teaching of mathematics and the natural sciences.

(2) To promote the interests of teachers of mathematics and the natural sciences.

(3) To foster the organization of such teachers in groups of various scopes.

(4) To support a journal or journals devoted to the aims of the society.

ARTICLE III. MEMBERSHIP.

The membership of any association representing the teachers of mathematics, the teachers of the natural sciences or the teachers of both [throughout one or more states or territories,]* shall be eligible to membership in this society. The application of any such association on be-

*It is the opinion of C. E. Comstock and F. T. Jones that this phrase should be omitted on the ground that sufficient safeguard is given membership by Art. V, paragraph 3 of this constitution.

half of its members shall be made through its council or executive committee, and shall be submitted to the vote of the council of the society.

ARTICLE IV. DUES.

Each association whose members are admitted to the society shall pay annual dues in proportion to the number of its members admitted to the society.

ARTICLE V. OFFICERS.

The officers of the society shall be a president, two vice-presidents, a treasurer and a secretary.

The council of the society shall consist of these officers and other elected councilors. Each co-operating association shall be entitled to elect one councilor for every one hundred active members, provided, however, that each association may elect one councilor and that no association may elect more than five councilors.

It shall be the duty of the council to pass on applications for membership, to arrange for all meetings of the society, to appoint committees and to transact all business of the society arising between meetings of the society.

ARTICLE VI. MEETINGS.

The society shall hold a meeting annually and such meeting shall be held at the same time and place as the National Educational Association unless the council shall direct otherwise.

ARTICLE VII. ELECTIONS.

The officers of the society shall be elected by ballot at the annual meeting of each year. An official ballot shall be sent to each member one month prior to the annual meeting and such ballot, if returned to the secretary in an envelope bearing the name of the voter, shall be counted at the annual meeting. Each such ballot shall contain a name proposed by the council for each vacancy with blank spaces in which the voter may substitute other names. A majority of all votes cast in person or by mail shall be necessary to election. In case of failure to secure a majority for any office the members present at the annual meeting shall choose by ballot between the two having the highest number of votes. The ex-presidents of the society shall be permanent members of the council; the term of office otherwise shall be one year. Aftr the annual meeting in 1908 for election as president shall be eligible those members of the society who have served in the office of vice-president and who are not holding offices as president or vice-president; and for election as vice-president shall be eligible those members of the society not holding offices as president or vice-president.

If the president of the society die or resign before the expiration of his term of office, the council may designate one of the vice-presidents to serve as acting president until the next annual meeting when a president shall be elected by the society. Such vacancies as may exist at any time among the other officers shall be filled by the council.

ARTICLE VIII. AMENDMENTS.

The constitution may be amended at any regular meeting of the Society by a two-thirds vote of the members present, provided that the pro-

posed change has been recommended by the council and has been announced in the call for the meeting sent out at least one month in advance of the meeting. Otherwise, an amendment of the constitution proposed at a meeting of the society cannot be acted upon until the meeting of the following year, when it must receive the favorable votes of two-thirds of the members present. Fifty members shall constitute a quorum for the consideration of a change of the constitution.

(Signed) THOMAS S. FISKE, Chairman.
FRANKLIN T. JONES, Secretary.

RESOLUTIONS ADOPTED BY THE CENTRAL ASSOCIATION OF SCIENCE AND MATHEMATICS TEACHERS DEC. 2, 1905.

Whereas: It is proposed to form an American Society of Science and Mathematics Teachers, and

Whereas: It is proposed to hold one annual meeting at the time of the N. E. A. and

Whereas: The proposed constitution of this new association does not meet the ideals of the Central Association of Science and Mathematics Teachers and

Whereas: The Central Association of Science and Mathematics Teachers believes that those associations should be in existence which will do the greatest good to the greatest number, and,

Whereas: We believe the Central Association of Science and Mathematics Teachers is doing its work in the Mississippi Valley, and

Whereas: The Central Association of Science and Mathematics Teachers is the largest and strongest body of Science and Mathematics Teachers in the United States, and

Whereas: The Central Association of Science and Mathematics Teachers believe in the unification or consolidation of the various science and mathematics associations in the country as speedily as possible; and

Whereas: In order to secure perfect harmony between all existing science and mathematics associations now existing and who desire to unite with a larger and stronger body.

Therefore, be it resolved, that the Central Association of Science and Mathematics Teachers recommend and urge the affiliations of all local organizations in a particular state with the state science and mathematics organization, if there be one, and that all state organizations unite with the Central Association of Science and Mathematics Teachers as soon as the several state constitutions can be amended to permit this action, in order that in our territory (Mississippi Valley) unification of work and unitedness of purpose may be secured. Also that we recommend a like action for the Atlantic and Pacific slopes. And, also be it resolved, that these three bodies unite into one American Science and Mathematics Teachers Associations, meeting once each year and that the three units also hold an annual meeting within their own territory.

And be it further resolved: That the Central Association of Science and Mathematics Teachers stands for the betterment of science and mathematics teaching in all its phases and will do all in its power to accomplish this.

MEETINGS.

THE MISSOURI SOCIETY OF TEACHERS OF MATHEMATICS

Will meet at Jefferson City at the same time as the State Teachers' Association, which convenes Dec. 27-29. The program will occupy two afternoons, one of which will be devoted mainly to the high school and one mainly to the elementary school. A complete program, constitution and list of members will be sent out two weeks before the meeting to members and to all others who request it. The following is a partial list of papers to be presented:

Maxima and Minima, George R. Dean, School of Mines, Rolla;

Laboratory Methods in Algebra Teaching, Oliver E. Glenn, Drury College, Springfield.

Some Problems of Arithmetic in the Grades, Thos. P. Jandon, Webster School, Kansas City.

Paper, The Treatment of Limits in Elementary Geometry, Albert M. Wilson, McKinley High School, St. Louis.

What Should Be Emphasized in the Teaching of Arithmetic, J. H. Scarborough, State Normal, Warrensburg.

Discussion, J. A. Whiteford, St. Joseph.

What Should Be Taught in Arithmetic and What Omitted?

U. S. Hall, Glasgow.

J. M. Greenwood, Kansas City.

E. R. Hedrick, Columbia.

Class exhibition—what can be done by mental drilling in arithmetic? U. S. Hall, Glasgow.

L. D. AMES, Secretary.

CENTRAL ASSOCIATION OF SCIENCE AND MATHEMATICS TEACHERS.

The fifth annual meeting of the central Association of Science and Mathematics Teachers was held in the Central Y. M. C. A. Building, Chicago, on Friday and Saturday, Dec. 1 and 2, 1905.

The general session on Friday forenoon was held in the Auditorium on the second floor. After being called to order by President Caldwell, Mr. Walter M. Wood, Director of Educational Work in the Y. M. C. A. of Chicago, gave an address of welcome.

President Caldwell announced the following committees: Nomination—J. H. Smith, Chicago; S. A. Douglas, St. Louis, and Miss Elma Chandler, Elgin. Auditing Committee—W. W. Whitney, Chicago; Grant Smith, Chicago; Miss Mabel Sykes, Chicago.

The first address of the morning was given by Dr. T. C. Chamberlin, Head Professor of Geology, University of Chicago. Subject: "Hypothesis of the Origin of the Earth." He outlined his recently announced "accretion theory" of the earth's origin, which, if accepted by scientists, will overthrow the so-called "nebular hypothesis" and do away with most of the ideas commonly held as to the origin of the earth and the other planets.

Prof. Chamberlin, who is one of the foremost living geologists, has been working since 1896 at his theory, which will be given to the public in the second volume of his work on geology, which is to be published

in January. Scientists all over the world have watched Prof. Chamberlin's work with the keenest interest.

According to the "nebular hypothesis," or LaPlace theory, which has been held for over a century, the earth and the other planets originally were masses of molten matter thrown off from the sun by its whirling motion and gradually cooled to their present condition. This theory Prof. Chamberlin attacks on the assumption that the sun never had a whirling velocity sufficient to throw off such bodies.

"The mechanics of the solar system don't work right," he said. "Every body has a momentum which is constant as long as its size does not change. To throw off Neptune the sun would have to have a momentum 200 times as great as it has now, and to throw off the earth one 1,800 times as great. Another difficulty with the old theory is that according to it all the satellites of a body should be revolving in the same direction. A ninth satellite of Saturn has been recently discovered, and it is going the wrong way!"

Prof. Chamberlin's theory is that a disturbance was caused in our sun by the near approach of, or collision with, some other sun, which drew off projections of nebulous matter from our sun and gave them a rotary motion by its own attraction as it passed. These particles of matter then gradually gathered together to form the planets.

The internal heat of the earth, says Prof. Chamberlin, developed by compression as the body grew. The theory that the moon once had an atmosphere, he asserts, must also be given up.

The second address was given by Ernest A. Stirling, U. S. Department of Forestry. Subject: "Relation of Forestry to Public School Instruction."

This will be published in a succeeding number of SCHOOL SCIENCE AND MATHEMATICS.

The afternoon was devoted to section meetings reports of which appear elsewhere or in a later number of the magazine.

The business session was held in the lecture room on the second floor, Saturday at 9 a. m. The Secretary reported a most successful year. One state association, the Indiana Science Teachers Association, had affiliated and its members had thereby become members of the Central Association of Science and Mathematics Teachers. Negotiations were in progress with other state associations tending toward affiliation.

Seventy-two applicants for membership were recommended by the executive committee and were unanimously elected on payment of dues.

W. E. Tower, chairman of the committee on local centres, reported the formation of a Local Centre at Jacksonville, Ill., Nov. 23, 1905, with 12 members, with the following officers:

President, Wm. E. Beal, Illinois College; Vice President, Principal Hopkins, Brown's Business College; Secretary, Miss Ione Kenchler, High School. Additional members of Executive Committee, Miss Anderson, Illinois College for Women; Mr. Maddigan, Routt College.

C. E. Comstock, chairman of the committee to attend the conference at Asbury Park, July 5, 1905, made a report as to the action taken con-

cerning the proposed American Association of Mathematics and Science Teachers. This was discussed by Messrs. Upham, Whitney, Comstock, Turton and Caldwell. C. H. Smith offered a resolution stating the attitude of the Central Association of Science and Mathematics Teachers toward the new association. This was unanimously adopted. The resolution appears on another page.

President Caldwell in a few well chosen remarks strongly urged all present to do missionary work for the association and SCHOOL SCIENCE AND MATHEMATICS.

The auditing committee made a partial report. Their final report was ordered printed in the magazine.

The following amendment to the constitution was adopted: That past presidents be *ex officio* members of the Executive Committee.

The nominating committee reported as follows: President, O. W. Caldwell, Charleston, Ill.; Treasurer, C. W. D. Parsons, 320 Main St., Evanston, Ill.

The report was adopted and they were unanimously elected. The committee on Local Centres: W. E. Tower, Chicago; G. P. Knox, St. Louis, Mo., and F. T. Jones, Cleveland, Ohio, were reappointed for another year.

J. E. Armstrong, chairman of the Committee on Temperance, presented the following resolution which was adopted:

Whereas, we believe that the teachers of science are in practical accord with your committee in disapproving of the general scheme of trying to teach temperance through a scientific knowledge of Physiology and believing that temperance is a moral rather than a physiological question and believing that the ends of both temperance and education would be better served by a more rational course of study based upon hygiene rather than upon Physiology, therefore be it

Resolved: That the secretary of this association be instructed to send a request to the executive committees of the State Teachers' Associations of each of the states represented in this association to appoint a committee of six representative teachers to prepare a course of study in hygiene which shall recognize that the great bodies of teachers in our states are among the leaders of moral and scientific education of the children; a course of study that shall not require our teachers to violate the laws of teaching or their own consciences; and be it further

Resolved: That we commend the action of the teachers of Massachusetts in securing the co-operation of the temperance organizations of that state in preparing the course of study issued by the committee of twelve of that state, and recommend that when these courses of study are adopted steps be taken to have them adopted by the legislatures instead of the present laws.

C. W. D. Parsons, offered a resolution regarding the preservation of Niagara Falls as a science park which was adopted.

A vote of thanks was extended to the officers of the Y. M. C. A. for the use of the building.

Prof. T. C. Chamberlin and Mr. Ernest A. Stirling were made honorary members of the association. After announcements of excursions the association adjourned for section meetings.

It was the opinion of all present that this was the best attended and most successful meeting in the history of the association, nearly 500 teachers being in attendance. One teacher, who had come 300 miles, said the address of Prof. Knipp and Dr. Kahlenberg Saturday morning, in the joint session of the Chemistry and Physics section, alone more than repaid him for the trip.

One of the most attractive features was the very complete exhibit of apparatus. Ten of the largest apparatus firms in America made exhibits. The Biology and Earth Science sections had displays of home made apparatus, note books, etc.

C. M. TURTON,
Secretary.

The Chemistry Section assembled at 2 p. m., Friday, in Room No. 2, seventh floor, about twenty-five were present. Dr. Jones, of the University of Chicago, made an address on Recent Discoveries in Organic Chemistry, in which he discussed the effect of recent investigations on the theories of atoms, valency and ions. Mr. F. J. Watson of the Wm. McKinley High School, Chicago, read a paper on The Equipment of a Chemical Laboratory in the Secondary School, which was devoted chiefly to description of that in the McKinley High School, and which was followed by a general discussion. Mr. F. C. Irwin, of Detroit, then read a paper on The Reference Library in Chemistry, and Mr. A. B. Crowe, Charleston, Ill., spoke on the same subject. On motion, the Chair appointed a committee consisting of Mr. Irwin, Dr. Jones, and Miss Jennie Caplin, of Minneapolis, to prepare a list of reference books to be recommended for use in secondary schools.

At the Saturday morning session officers were elected for the ensuing year as follows:

Chairman, F. J. Watson, Wm. McKinley High School, Chicago.

Vice Chairman, W. C. Hawthorne, Association College, Chicago.

Secretary, A. B. Crowe, State Normal School, Charleston, Ill.

After which the section adjourned to Room 1, where the joint session with the Physics Section, they listened to the address of Dr. Kahlenberg on Recent Discoveries in Osmosis.

THE COLORADO MATHEMATICS SOCIETY.

The Colorado Mathematics Society, having for its main object the improvement of the teaching of mathematics, was organized in Denver, Colo., on Dec. 2, 1904. Professor DeLong of the University of Colorado was elected president, and Mr. Smith of the North Denver High School, secretary. The next meeting will be held on January 6, 1906. SCHOOL SCIENCE AND MATHEMATICS was adopted as the official organ.

The tenth annual meeting of the NEW YORK STATE SCIENCE TEACHERS' ASSOCIATION will be held at the High School, Syracuse, N. Y., December 27-29, 1905. The attention of teachers of mathematics is called to the fact that a section for MATHEMATICS has been formed. A fine program has been prepared.

BOOK REVIEWS.

Geology. Thomas C. Chamberlain and Rolin D. Salisbury. Vol. I, Geologic Processes and their Results, Henry Holt & Co.

The success of this book is evidenced by the necessity of a second edition so soon. The secondary school teacher can never hope to use a book of this size in his classes but it serves a need long felt for a reference work to which he can send the student with a certainty that he will read with an increased interest. Frequently a book of fundamentals makes no more appeal to the student than a dictionary and one whose task it is to arouse the first interest in the subject dreads their influence. It is a happy combination that permits scientific facts to be so presented that even those first entering the subject are interested.

The subject of this first volume makes it a most valuable adjunct to those teaching physical geography as well as geology. The plates and maps are wisely selected and the printing of the topographic maps in the colors of regular maps helps the student materially.

It is hardly probable that any secondary school teacher would attempt to follow the book in the proposed quantitative system of classification of the rocks. Certainly it is hoped that none would attempt to impose such upon beginners. The proposed field system is very easily handled and nearly all that is necessary.

The book has passed beyond the stage of recommendation and prophecy and has already won its place on the shelves of every geological library, however small. E. C. Case.

Methods in Plant Histology. By Charles J. Chamberlain. 272 pages, 8vo cloth, net $2.25, postpaid $2.39. The University of Chicago Press.

This is the only complete work thus far issued on the preparation of botanical material for the microscope. It is adapted to the needs of the solitary worker with limited apparatus as well as to college classes. Freehand sectioning, the paraffin method, the celloidin method, and the glycerine method are treated in detail. Later chapters give directions for making such preparations as are needed in the study of the various groups of the plant kingdom. Formulas are given for reagents. The new edition includes a description of Professor Klebs's method for securing reproductive phases in the Algae and Fungi, and chapters on the Venetian turpentine method, microchemical tests, freehand sectioning, special methods, and the use of the microscope. The volume is thus increased in bulk from 168 to 272 pages.

BOOKS RECEIVED.

An Elementary Laboratory Course in Chemistry. Frank B. Kenrick, University of Toronto, and Ralph E. DeLacy, University of Toronto. Morang & Co., 90 Wellington St. W., Toronto. Price $1.00. 90 pages.

FROBLEM DEPARTMENT.

PROFESSOR IRA M. DELONG,
University of Colorado, Boulder, Colo.

Readers of the Magazine are invited to send solutions of the problems in this department and also to propose problems in which they are interested. Solutions and problems will be duly credited to the author. Address all communications to Ira M. DeLong, Boulder, Colo.

ALGEBRA.

4. *Proposed by S. Epsteen, Boulder, Colo.*

To complete a certain work, A requires m times as many days as B and C together; B requires n times as many as A and C together; C requires p times as many as A and B together; compare the times in which each would do it, and prove that

$$\frac{1}{m+1}+\frac{1}{n+1}+\frac{1}{p+1}=1.$$

Solution by I. L. Winckler.

Denoting the number of days which A takes to do the work alone, by A, the number which B takes by B and the number which C takes by C, then $\frac{1}{A}$, $\frac{1}{B}$, $\frac{1}{C}$ represent the fractions of the work which each one, working alone, will do in one day. From the conditons of the problem it follows at once that

$$(1)\ldots\ldots\frac{m}{A}=\frac{1}{B}+\frac{1}{C};\ (2).\ \ldots\ldots\frac{n}{B}=\frac{1}{A}+\frac{1}{C};\ (3)\ldots\ldots\frac{p}{C}=\frac{1}{A}+\frac{1}{B}.$$

Subtracting (2) from (1) and (3) from (2) there result, by easy reduction,

$$mB-nA=A-B,\ nC-pB=B-C$$

and therefore $A : B : C=m+1 : n+1 : p+1$.

From equation (1) we see that $m+1=\frac{AB+AC+BC}{BC}$, and similarly, from (2) and (3) $n+1=\frac{AB+AC+BC}{AC}$, $p+1=\frac{AB+AC+BC}{AB}$. Therefore

$$\frac{1}{m+1}+\frac{1}{n+1}+\frac{1}{p+1}=1.$$

5. *Proposed by the Editor.*

Find the rational prime factor of x^9-y^9.

Solution by P. S. Berg, Larimore, N. D.

$x^9-y^9=(x^3)^3-(y^3)^3=(x^3-y^3)\ (x^6+x^3y^3-y^6)=(x-y)\ (x^2+xy+y^2)\ (x^6+x^3y^3+y^6)$*

Also solved by Emma Hyde, I. L. Winckler, W. F. Sloan.

6. *Proposed by Mary C. Tracy, Passaic, N. J.*

A man is twice as old as his wife was when he was as old as his wife

*It should be demonstrated that $x^6+x^3y^2+y^6$ is prime in the domain of integers.—*Ed.*

now is; when she is as old as he now is, the sum of their ages will be one hundred years. Find their ages.

Solution by John Kaiser, Marietta, Ohio.

Let x, y denote the ages of the wife and husband, then from the first condition $y=2\,[x-(y-x)]=4x-2y$ or $3y=4x$ ..... (1)

From the second condition

$x+(y-x)+[y+(y-x)]=100$ or $3y-x=100$ ..... (2)

Solving (1) and (2), we find $x=33\frac{1}{3}$, $y=44\frac{4}{9}$.

Also solved by M. R. Beck, P. S. Berg, Carl Swartz, H. H. Seidell, Emma Hyde, E. L. Brown, I. L. Winckler, W. F. Sloan. One incorrect solution was received.

The following additional solutions were received too late to be credited in the December number: Algebra No. 1, M. R. Beck, Cleveland, Ohio; Algebra No. 2, P. S. Berg, Larimore, N. D.; Algebra No. 3, M. R. Beck and P. S. Berg.

GEOMETRY.

1. In any triangle, ABC, if BP, CQ be drawn perpendicular to CA, BA, produced if necessary, then shall $\overline{BC}^2 = AB \cdot BQ + AC \cdot CP$.

Solution by P. S. Berg, Larimore, N. D.

$BC^2=AB^2+AC^2-2AC \cdot AP$,

$BC^2=AB^2+AC^2-2AB \cdot AQ$.

Adding, $2BC^2=2(AB^2+AC^2-AC \cdot AP-AB \cdot AQ)$,

$BC^2=AB^2+AC^2-AC \cdot AP-AB \cdot AQ=$

$AB\,(AB-AQ)+AC\,(AC-AP)=$

$AB \cdot BQ+AC \cdot CP$.

Also solved by H. H. Seidell, I. L. Winckler, Grant B. Grumbine, M. R. Beck, T. M. Blakslee, E. L. Brown.

3. *Proposed by the Editor.*

In a given triangle inscribe another whose sides shall pass through given points.

Solution.

No solution by elementary methods having been received, the following one, based on the theory of homographic divison, is offered. Let ABC be the given triangle, PQR the given points (outside of the triangle) the first lying in the angle C, the second in the angle A, the third in the angle B. Take on AC any arbitrary point D. Join PD, intersecting AB in E; join EQ, intersecting BC in F; lastly, join FR, intersecting AC in D. The two points, D, D, evidently form two homographic divisions on AC, the double points of which will be vertices of two triangles satisfying the required conditions.

4. *Proposed by H. B. Leonard, Boulder, Colo.*

If perpendiculars AP, BQ, CR be drawn from the angular points of a triangle ABC upon the sides, show that they will bisect the angles of the triangle PQR.

Solution by the Proposer.

Let ABC be any triangle, and AP, BQ and CR the three perpendiculars. Join P and R, P and Q, and R and Q.

In the right triangles AQB and ARC, the angle BAC is common to both. Therefore they are similar. Hence AB:AC—AQ . AR. The triangles BAC and RAQ have the angle RAQ common and the including sides proportional. Therefore they are similar, and the angle ARQ=the angle ACB. In the same manner we may prove the angle PRB=the angle ACB; and therefore the angle ARQ=the angle PRB. Angle CRA—angle QRA=angle CRB—angle PRB. Hence, angle QRC=angle PRC, and the angle QRP is bisected by the perpendicular CR. In a similar manner it can be proved that AP bisects the angle RPQ, and BQ bisects the angle RQP.

Also solved by P. S. Berg, Colborne Crsselman.

5. *Proposed by S. Epsteen, Boulder, Colo.*

A common tangent is drawn to two circles which touch externally; if a circle be described on that part of it which lies between the points of contact, as diameter, this circle will pass through the common point of contact of the two circles, and be touched by the line which joins their centers.

Solution by P. S. Berg, Larimore, N. D.

Let O and O be two circles tangent at A; with CB as a common external tangent. Draw BA, CA, then the angle BAC is a right angle.* Therefore the circumference of a circle whose diameter is BC passes through A and is tangent to OO.

Also solved by Grant B. Grumbine, Josiah Bartlett, H. H. Seidell, W. F. Sloan, E. L. Brown, I. L. Winckler.

6. *Proposed by Mary C. Tracy, Passaic, N. J.*

The circumference of a circle can be divided into five equal parts as follows: Draw two perpendicular diameters AC, DE, meeting at the center B. Bisect AB in F, with F as a center and FD as radius describe a circle meeting AC in G. Then DG is a chord of one-fifth the circumference. Can this construction be proven geometrically?

Solution by M. R. Beck and Grant B. Grumbine.

We know that the side of a regular pentagon in terms of the radius is $\frac{1}{2}R\sqrt{10-2\sqrt{5}}$. [Wentworth, Revised, page 241, exercise 463.]

Since BD=BC=r, FB=$\frac{1}{2}$r; $\overline{FD}^2=r^2+\frac{1}{4}r^2$.

$\therefore$ FD=$\frac{r}{2}\sqrt{5}$=FG.

BG=FG−FB.

$=\frac{r}{2}\sqrt{5}-\frac{1}{2}r.$

$$\overline{DG}^2=r^2+\left[\frac{r}{2}\sqrt{5}-\frac{1}{2}r\right]^2$$

$$=\frac{5}{2}r^2-\frac{r^2}{2}\sqrt{5}$$

$$\overline{DG}=\sqrt{\frac{5}{2}r^2-\frac{r^2}{2}\sqrt{5}}$$

$$=\frac{r}{2}\sqrt{10-2\sqrt{5}}$$

Q. E. D.

*Joining the middle point M of CB to the point of contact A, we see that MC=MA=MB and therefore CAB is a right angle.—Ed.

Also solved by P. S. Berg. Some geometric solutions which arrived late will be printed next month.

One incorrect solution was received.

A solution of Geometry No. 2 was received from M. R. Beck, Cleveland, Ohio, too late for crediting in the December number.

TRIGONOMETRY.

3. *Proposed by H. B. Leonard, Boulder, Colo.*

Prove that $\tan^{-1}\frac{1}{7}+2\tan^{-1}\frac{1}{3}=45^{0}$.

Solution by Orlando S Stetson, Syracuse Univ., Syracuse, N. Y.

Let $\tan^{-1}\frac{1}{7}+2\tan^{-1}\frac{1}{3}=x$, then

$\tan x=\tan\left(\tan^{-1}\frac{1}{7}+2\tan^{-1}\frac{1}{3}\right)$, and

$$\tan x=\frac{\frac{1}{7}+\frac{\frac{2}{3}}{1\cdot\left(\frac{1}{3}\right)^{2}}}{1-\frac{\frac{1}{7}\cdot\frac{2}{3}}{1-\left(\frac{1}{3}\right)^{2}}}=1$$

Therefore $x=n\pi+\frac{\pi}{4}$, and

$x=45$ when $n=0$.

Also solved by Grant B. Grumbine, H. H. Seidell, W. D. Higden, I. L. Winckler, E. L. Brown, P. E. Graber, and the proposer.

PROBLEMS FOR SOLUTION.

Suggestions to Problem Solvers.

1. Write legibly, being specially particular as to proper names.

2. Write on one side of the paper only, reserving at the top and the left a two-inch margin for the use of the editor.

3. It is very desirable that each problem be given a separate sheet; also, a separate sheet for any communication not intended for publication.

4. In any algebraic solution, or system of equations, only those should be numbered to which subsequent references are made.

5. The simpler geometrical solutions should be so worded that they may be readily understood without the aid of a diagram. If a diagram can not well be omitted, it should be furnished to the editor in duplicate, one copy on the sheet with the solution, and another copy on a separate sheet, done accurately in ink so that it may be photographed.

6. All solutions should be in the following standard form:

(a) The subject heading (Algebra, Geometry, Trigonometry).

(b) The serial number of the problems in Hindoo figures.

(c) The name, academic degrees, and postoffice address of the contributor.

(d) The body of the solution.

ALGEBRA.

1. A and B start to run a race to a certain post and back again. A returning meets B at 90 yards from the post and arrives at the starting-place 3 minutes before him. If he had returned immediately to meet B,

he would have met him at one-sixth of the distance between the post and the starting-place. Find the length of the course and the duration of the race.

2. Solve $\frac{(1+x)^{\frac{1}{2}}-1}{(1-x)^{\frac{1}{2}}+1}+\frac{(1-x)^{\frac{1}{2}}+1}{(1+x)^{\frac{1}{2}}-1}=a$

3. Reduce into simple or partial fractions

$$\frac{6x^2-4x-6}{(x-1)\ (x-2)\ (x-3)}$$

GEOMETRY.

4. From a given point between two given straight lines draw a straight line which shall be terminated by the given lines and bisected at the given point.

5. If ABC be a triangle with the angles B. C, each double of the angle A, prove that $\overline{AB}^2=\overline{BC}^2+AB,\ BC$.

6. If from the angle A of any parallelogram any line be drawn cutting the diagonal in P and the sides BC, CD in Q and R, show that $\overline{AP}^2=PQ \bullet PR$.

TRIGONOMETRY.

7. Prove that $\sin x+\sin y+\sin z=4\sin\frac{x+y}{2}\sin\frac{x+z}{2}\sin\frac{y+z}{2}+\sin(x+y+z)$

PUBLISHERS' NOTICE.

The present indications are that after January 1 the printers' strike will be more wide-spread than at present.

Should there be any delay in receiving the February number, we trust that our readers will be patient.

The Chicago Clearing House has just put into effect a charge of 10 cents for collecting checks for less than ten dollars, and 15 cents for larger checks. If personal checks are sent, please add this 10 cents. Please remit by money order, express or postal, or by bank draft.

SCIENCE AND MATHEMATICAL SOCIETIES.

Under this heading is published each month the name and officers of such societies as furnish this information.

Central Association of Science and Mathematics Teachers.

President, Otis W. Caldwell, State Normal School, Charleston, Ill.
Secretary, Chas. M. Turton, 440 Kenwood Terrace, Chicago, Ill.
Treasurer, Chas. W. D. Parsons, 320 Main St., Evanston, Ill.

Annual meeting Friday and Saturday immediately following Thanksgiving.

Chicago Center, C. A. S. and M. T.

President, W. C. Hawthorne, Central Y. M. C. A., Chicago.
Vice-President, P. B. Woodworth, Lewis Institute, Chicago.
Secretary, C. E. Osborne, High School, Oak Park, Ill.

North-Eastern Ohio Center, C. A. S. and M. T.

President, C. W. Sutton, Central High School, C eveland.
Vice-President, Miss J. Cora Bennett, East High School, Cleveland.
Secretary-Treasurer, J. E. Crabbe, Glenville High School, Cleveland

Association of Ohio Teachers of Mathematics and Science.

President, William McPherson, Ohio State University, Columbus.
Vice-President, Franklin T. Jones, University School, Cleveland.
Secretary, Thomas E. McKinney, Marietta College, Marietta.

Science Section of the Michigan School Masters' Club.

Chairman, H. M. Randall, Ann Arbor.
Vice-Chairman, F. C. Irwin, Central High School, Detroit.
Secretary, DeForrest Ross, Ypsilanti.

WANTED!

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